CAPÍTULO 15

Exercícios 15.1

1. ***b***) $f(x, y) = 2x^2 - 3y^2 + xy\, f(4, 3) = 17$ e $f(1, 2) = -8$

$$\nabla f\ (\bar{x}, \bar{y}) = \left(\frac{\partial f}{\partial x}(\bar{x}, \bar{y}), \frac{\partial f}{\partial y}(\bar{x}, \bar{y})\right) = (4\bar{x} + \bar{y}, -6\bar{y} + \bar{x})$$

Pelo T.V.M., $f(4, 3) - f(1, 2) = \nabla f\ (\bar{x}, \bar{y})\ [(4, 3) - (1, 2)]$

Segue que $13\bar{x} - 3\bar{y} = 25$. Mas $(\bar{x}, \bar{y}) = (1, 2) + t(3, 1)$ com $1 < \bar{x} < 4$

Logo, $\bar{x} - 3\bar{y} = -5$. Resolvendo o sistema $\begin{cases} 13\bar{x} - 3\bar{y} = 25 \\ \bar{x} - 3\bar{y} = -5 \end{cases}$

Temos $\bar{x} = \dfrac{5}{2}$ e $\bar{y} = \dfrac{5}{2}$. Portanto, $\bar{P} = \left(\dfrac{5}{2}, \dfrac{5}{2}\right)$.

Exercícios 15.3

1. ***b***) $\dfrac{\partial f}{\partial x} = y \cos xy + 3x^2 - y = P$

$$\frac{\partial f}{\partial y} = x \cos xy - x + 3y^2 = Q$$

$$\frac{\partial Q}{\partial x} = -xy \operatorname{sen} xy + \cos xy - 1 = \frac{\partial P}{\partial y}.$$

Portanto, $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ (condição necessária verificada: o sistema pode admitir soluções).

Integrando-se a 1.ª equação em relação a x, mantendo y constante, a função $\operatorname{sen} xy + x^3 - xy$ é solução da 1.ª equação.

Integrando-se a 2.ª equação em relação a y, mantendo-se x constante, a função $\operatorname{sen} xy - xy + y^3$ é solução da 2.ª equação.

Logo, $f(x, y) = \operatorname{sen} xy + x^3 - xy + y^3 + k$ é a solução do sistema.

2. $\dfrac{\partial f}{\partial x} = 2xy^3 - 2x = P$ ① $\quad \dfrac{\partial P}{\partial y} = 6xy^2$

$\dfrac{\partial f}{\partial y} = 3x^2y^2 + 2y - 1 = Q$ ② $\quad \dfrac{\partial Q}{\partial x} = 6xy^2$

$\left.\begin{matrix} \dfrac{\partial P}{\partial y} = 6xy^2 \\ \dfrac{\partial Q}{\partial x} = 6xy^2 \end{matrix}\right\}$ condição necessária para que o sistema possa admitir solução

$x^2y^3 - x^2$ é solução de ① e $x^2y^3 + y^2 - y$ é solução de ②

Portanto, $f(x, y) = x^2y^3 - x^2 + y^2 - y + k$ é a solução do sistema.
$f(1, 2) = 7 \Rightarrow 9 + k = 1 \Rightarrow k = -8.$

Logo, $f(x, y) = x^2y^3 - x^2 + y^2 - y - 8.$

4. $\dfrac{\partial f}{\partial x} = x^2 + y^2 + 1 = P \quad \dfrac{\partial P}{\partial y} = 2y$

$\dfrac{\partial f}{\partial y} = x^2y^2 + 1 = Q \quad \dfrac{\partial Q}{\partial x} = 2x$

$\left.\begin{matrix} \dfrac{\partial P}{\partial y} = 2y \\ \dfrac{\partial Q}{\partial x} = 2x \end{matrix}\right\} \neq$

Não existe $f\colon \mathbb{R}^2 \to \mathbb{R}$ pois, $\dfrac{\partial P}{\partial y} \neq \dfrac{\partial Q}{\partial x}$.

5. $\nabla \varphi_1\ (x,\ y) = \left(-\dfrac{y}{x^2 + y^2},\ \dfrac{x}{x^2 + y^2}\right)$

$$\left.\begin{matrix} \dfrac{\partial \varphi_1}{\partial x} = -\dfrac{y}{x^2 + y^2} = P \ \Rightarrow\ \dfrac{\partial P}{\partial y} = \dfrac{y^2 - x^2}{(x^2 + y^2)^2} \\ \dfrac{\partial \varphi_1}{y} = \dfrac{x}{x^2 + y^2} = Q \ \Rightarrow\ \dfrac{\partial Q}{\partial x} = \dfrac{y^2 - x^2}{(x^2 + y^2)^2} \end{matrix}\right\rangle \text{(Condição necessária)}$$

Integrando-se a 1.ª equação em relação a x, mantendo y constante:

$$\int -\frac{y\, dx}{x^2 + y^2} = -\int \frac{dx}{y\left(1 + \left(\dfrac{x}{y}\right)^2\right)} = -\text{arctg}\ \frac{x}{y} + k.$$

Analogamente $\displaystyle\int \frac{x\, dy}{x^2 + y^2} = -\int \frac{\left(-\dfrac{x}{y^2}\right) dy}{1 + \left(\dfrac{x}{y}\right)^2} = -\text{arctg}\ \frac{x}{y} + k.$

$\varphi_1\ (x,\ y) = -\text{arctg}\ \dfrac{x}{y} + k,\ y > 0.$

$\varphi_1(1,\ 1) = \dfrac{\pi}{4}\ \Rightarrow\ -\text{arctg}\ 1 + k = \dfrac{\pi}{4}\ \Rightarrow\ -\dfrac{\pi}{4} + k = \dfrac{\pi}{4} \Rightarrow k = \dfrac{\pi}{2}$

Portanto< $\varphi_1\ (x,\ y) = -\text{arctg}\ \dfrac{x}{y} + \dfrac{\pi}{2},\ y > 0.$

6. $\nabla\varphi_2\ (x,\ y) = \left(-\dfrac{y}{x^2 + y^2},\ \dfrac{x}{x^2 + y^2}\right),\ x < 0,$ e $\varphi_2\ (-1,\ 1) = \dfrac{3\pi}{4}$. Temos

$$\int -\frac{y\ dx}{x + y^2} = \int \frac{\left(-\dfrac{y}{x^2}\right) dx}{1 + \left(\dfrac{y}{x}\right)^2} = \text{arctg}\ \frac{y}{x} + k$$

$$\int \frac{x\ dy}{x^2 + y^2} = \int \frac{dy}{x\left(1 + \left(\dfrac{y}{x}\right)^2\right)} = \text{arctg}\ \frac{y}{x} + k$$

$\varphi_2\ (x,\ y) = \text{arctg}\ \dfrac{y}{x} + k$

$\varphi_2\ (-1,\ 1) = \underbrace{\text{arctg}\ (-1)}_{-\frac{\pi}{4}} + k = \dfrac{3\pi}{4}\ \Rightarrow\ k = \pi$

Portanto, $\varphi_2\ (x,\ y) = \text{arctg}\ \dfrac{y}{x} + \pi,\ x < 0.$

7. Por **5**: $\varphi_1\ (x,\ y) = -\text{arctg}\ \dfrac{x}{y} + k_1,\ y > 0$

Por **6**: $\varphi_2\ (x,\ y) = \text{arctg}\ \dfrac{y}{x} + k_2,\ x < 0.$

Sabemos que $\varphi\ (-1,\ 1) = \dfrac{3\pi}{4}$

Então,

$-\text{arctg}\ (-1) + k_1 = \dfrac{3\pi}{4} \Rightarrow \dfrac{\pi}{4} + k_1 = \dfrac{3\pi}{4} \Rightarrow k_1 = \dfrac{\pi}{2}$

$\text{arctg}\,(-1) + k_2 = \dfrac{3\pi}{4} \;\Rightarrow\; k_2 = \dfrac{3\pi}{4} + \dfrac{\pi}{4} \;\Rightarrow\; k_2 = \pi.$

Portanto,

$$\varphi(x, y) = \begin{cases} -\text{arctg}\,\dfrac{x}{y} + \dfrac{\pi}{2} & \text{se } \; y > 0 \\ \text{arctg}\,\dfrac{y}{x} + \pi & \text{se } \; x < 0 \end{cases}$$

8. *a*) $\vec{F}\,(x, y) = x\,\vec{i} + y\,\vec{j}$, onde $P\,(x, y) = x$ e $Q\,(x, y) = y$. Temos

$$\frac{\partial P}{\partial y} = 0 \;\text{ e }\; \frac{\partial Q}{\partial x} = 0$$

O campo de forças $\vec{F}$ admite a função potencial $\varphi\,(x, y) = \dfrac{x^2}{2} + \dfrac{y^2}{2}$. Logo $\vec{F}$ é conservativo.

***d*)** $\vec{F}\,(x, y) = \dfrac{x\,\vec{i} + y\,\vec{j}}{(x^2 + y^2)^{3/2}} = \dfrac{x}{(x^2 + y^2)^{3/2}}\,\vec{i} + \dfrac{y}{(x^2 + y^2)^{3/2}}\,\vec{j}.$

$$\left.\begin{array}{l} \dfrac{x}{(x^2 + y^2)^{3/2}} = P\,(x, y) \;\Rightarrow\; \dfrac{\partial P}{\partial y} = -\dfrac{3xy}{(x^2 + y^2)^{5/2}} \\ \dfrac{y}{(x^2 + y^2)^{3/2}} = Q\,(x, y) \;\Rightarrow\; \dfrac{\partial Q}{\partial x} = -\dfrac{3xy}{(x^2 + y^2)^{5/2}} \end{array}\right\rangle \Rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}.$$

$$\int \frac{x\,dx}{(x^2 + y^2)^{3/2}} = -(x^2 + y^2)^{-\frac{1}{2}} \quad \text{e} \quad \int \frac{y\,dy}{(x^2 + y^2)^{\frac{3}{2}}} = -(x^2 + y^2)^{-\frac{1}{2}}$$

$\vec{F}$ admite a função potencial $\varphi\,(x, y) = -\dfrac{1}{\sqrt{x^2 + y^2}}$, logo é conservativo.

11. *a*) $\vec{F}\,(x, y) = -6x\,\vec{i} - 2y\,\vec{j}$. Temos

$U\,(x, y) = 3x^2 + y^2 + k.$
$U\,(0, 0) = 0 \;\Rightarrow\; k = 0.$

Logo, $U\,(x, y) = 3x^2 + y^2$.

***d*)** $\vec{F}\,(x, y) = x\,\vec{i} - xy\,\vec{j}$. Temos

$\dfrac{\partial}{\partial y}\,(x) = 0 \;$ e $\; \dfrac{\partial}{\partial x}\,(-xy) = -y$ e daí

$$\frac{\partial}{\partial x}(x) \neq \frac{\partial}{\partial y}(-xy).$$

$\vec{F}$ não é conservativo. Portanto, não existe a função energia potencial associada ao campo $\vec{F}$.

13. ***a***) Seja $U(x, y) = 2x^2 + \frac{1}{2}y^2$. Temos

$$\vec{F} = -\nabla U = -\frac{\partial U}{\partial x}\vec{i} - \frac{\partial U}{\partial y}\vec{j}, \text{ ou seja,}$$

$$\vec{F} = -x\,\vec{i} - y\,\vec{j}.$$

b) Seja $\gamma(t) = (x(t), y(t))$ com $x(0) = 1$ e $y(0) = 1$. Temos:

$$\ddot{x} = -x \quad \text{e} \quad \ddot{y} = -y$$
$$\ddot{x} + x = 0 \quad \text{e} \quad \ddot{y} + y = 0.$$

$x(t) = A_1 \cos t + B_1 \operatorname{sen} t$ e $y(t) = A_2 \cos t + B_2 \operatorname{sen} t$. Logo,
$\gamma(t) = (A_1 \cos t + B_1 \operatorname{sen} t, A_2 \cos t + B_2 \operatorname{sen} t)$.

De $x(0) = 1$, segue que $A_1 = 1$.
De $y(0) = 1$, segue que $A_2 = 1$.

E mais,

$$\vec{v}(t) = \frac{d\gamma}{dt} \quad \Rightarrow \quad \vec{v}(t) = (-A_1 \operatorname{sen} t + B_1 \cos t, -A_2 \operatorname{sen} t + B_2 \cos t).$$

Temos $\vec{v}_0 = (1, 1)$. Então, $B_1 = -1$ e $B_2 = 1$.
Portanto, $\gamma(t) = (\cos t - \operatorname{sen} t, \cos t + \operatorname{sen} t)$.
De $x(t) = \cos t - \operatorname{sen} t$ e $y(t) = \cos t + \operatorname{sen} t$ segue que $x^2 + y^2 = 2$. Logo, a trajetória é a circunferência de centro na origem e raio $\sqrt{2}$:

$$\gamma(t) = \sqrt{2}\left(\cos\left(t + \frac{\pi}{4}\right), \operatorname{sen}\left(t + \frac{\pi}{4}\right)\right).$$

Exercícios 15.4

1. ***a***) $f(x, y) = e^{x + 5y}$ e $(x_0, y_0) = (0, 0)$.

Temos: $f(0, 0) = 1$,

$$\frac{\partial f}{\partial x} = e^{x+5y}, \qquad \frac{\partial f}{\partial x}(0, 0) = 1,$$
$$\frac{\partial f}{\partial y} = 5e^{x+5y} \quad \text{e} \quad \frac{\partial f}{\partial y}(0, 0) = 5.$$

Polinômio de Taylor

$$P_1(x, y) = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0)$$
$$= f(0, 0) + \frac{\partial f}{\partial x}(0, 0)x + \frac{\partial f}{\partial y}(0, 0)y.$$

Logo, $P_1(x, y) = 1 + x + 5y$.

2. *a*) Seja $f(x, y) = P_1(x, y) + E_1(x, y)$

onde $E_1(x, y) = \dfrac{1}{2}\left[\dfrac{\partial^2 f}{\partial x^2}(\bar{x}, \bar{y})\, x^2 + 2\,\dfrac{\partial^2 f}{\partial x \partial y}(\bar{x}, \bar{y})\, xy + \dfrac{\partial^2 f}{\partial y^2}(\bar{x}, \bar{y})\, y^2\right]$

com $(\bar{x}, \bar{y})$ interno ao segmento de extremidades $(0, 0)$ e (x, y).

Supondo $x + 5y < 1$ temos também $\bar{x} + 5\bar{y} < 1$. Assim, para todo (x, y), com $x + 5y < 1$, segue que $e^{\bar{x} + 5\bar{y}} < 2$. Logo, $e^{\bar{x} + 5\bar{y}} < 3$.

Temos $\dfrac{\partial^2 f}{\partial x^2} = e^{x + 5y}$, $\dfrac{\partial^2 f}{\partial y^2} = 25e^{x + 5y}$, e $\dfrac{\partial^2 f}{\partial x \partial y} = 5e^{x + 5y}$,

$\left|e^{x + 5y} - P_1(x, y)\right| = \left|E_1(x, y)\right|$ e

$$E_1(x, y) = \frac{1}{2}\left[e^{\bar{x} + 5\bar{y}}\, x^2 + 10e^{\bar{x} + 5\bar{y}} xy + 25e^{\bar{x} + 5\bar{y}} y^2\right].$$

Segue, considerando $e^{\bar{x} + 5\bar{y}} < 3$,

$$\left|E(x, y)\right| < \frac{1}{2} \cdot 3\,(x^2 + 10xy + 25y^2).$$

Logo,

$$\left|e^{x + 5y} - P_1(x, y)\right| < \frac{3}{2}(x + 5y)^2, \text{ para } x + 5y < 1.$$

***b*)** Para $x = 0{,}01$ e $y = 0{,}01$ tem-se $x + 5y < 1$.

$$\left|E_1(x, y)\right| < \frac{3}{2}(x + 5y)^2 = \frac{3}{2}(0{,}06)^2 = 0{,}54 \times 10^{-2} < 10^{-2}.$$

Portanto o erro é inferior a 10^{-2}.

3. $f(x, y) = x^3 + y^3 - x^2 + 4y$. Temos

$\left|f(x, y) - P_1(x, y)\right| = \left|E_1(x, y)\right|$, onde

$$\left|E_1\ (x,\ y)\right| = \frac{1}{2}\left[\frac{\partial^2 f}{\partial y^2}\ (\bar{x},\ \bar{y})\ (x-1)^2 + 2\ \frac{\partial^2 f}{\partial x \partial y}\ (\bar{x},\ \bar{y})(x-1)(y-1) + \frac{\partial^2 f}{\partial y^2}\ (\bar{x},\ \bar{y})(y-1)^2\right].$$

Temos $\dfrac{\partial f}{\partial x} = 3x^2 - 2x;\ \dfrac{\partial^2 f}{\partial x^2}\ (\bar{x},\ \bar{y}) = 6\bar{x} - 2;\ \dfrac{\partial f}{\partial y} = 3y^2 + 4;\ \dfrac{\partial^2 f}{\partial y^2}\ (\bar{x},\ \bar{y}) = 6\bar{y}$

$E_1\ (x,\ y) = \dfrac{1}{2}\left|(6\bar{x} - 2)\ (x-1)^2 + 6\bar{y}\ (y-1)^2\right|$

Se $|x - 1| < 1$, então $0 < x < 2$ e $0 < \bar{x} < 2$

Se $|y - 1| < 1$, então $0 < y < 2$ e $0 < \bar{y} < 2$

$|6\bar{x} - 2| \leqslant |6\bar{x}| + |-2| < 12 + 2$

Portanto,

$$|\overbrace{f\ (x,\ y) - P_1\ (x,\ y)}^{|E\ (x,\ y)|}| < \frac{1}{2}\left|14\ (x-1)^2 + 12\ (y-1)^2\right|$$

Assim, para todo (x, y) com $|x - 1| < 1$ e $|y - 1| < 1$, temos:

$|f(x, y) - P_1\ (x, y)| < 7\ (x-1)^2 + 6\ (y-1)^2$.

4. ***a***) $P_1\ (x, y) = f(1, 1) + \dfrac{\partial f}{\partial x}\ (1, 1)\ (x-1) + \dfrac{\partial f}{\partial y}\ (1, 1)\ (y-1)$,

$P_1\ (x, y) = 5 + (x-1) + 7\ (y-1)$. Logo,

$P_1\ (x, y) = x + 7y - 3$. Temos

$P_1\ (1{,}001,\ 0{,}99) = 4{,}931$ e daí

$f(x, y) \cong 4{,}931$.

b) $|E\ (x, y)| < 7\ (x-1)^2 + 6\ (y-1)^2 = 7\ (1{,}001 - 1)^2 + 6\ (0{,}99 - 1)^2$
$= 7 \cdot 10^{-6} + 6 \cdot 10^{-4} = 10^{-3}\ (0{,}7 \cdot 10^{-2} + 0{,}6) < 10^{-3}$.

6. Seja $f(x, y) = ax^2 + b\,xy + cy^2 + dx + ey + m$.

Temos:

$$f(x_0 + h, y_0 + k) - f(x_0, y_0) = \frac{\partial f}{\partial x}\ (x_0, y_0) \cdot h + \frac{\partial f}{\partial y}\ (x_0, y_0)\ k + E\ (h, k)$$

onde

$$E\ (h, k) = \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}\ (\bar{x},\ \bar{y})\ h^2 + 2\ \frac{\partial^2 f}{\partial x \partial y}\ (\bar{x},\ \bar{y})\ hk + \frac{\partial^2 f}{\partial y^2}\ (\bar{x},\ \bar{y})k^2\right].$$

Como (x_0, y_0) é ponto crítico de f: $\dfrac{\partial f}{\partial x}\ (x_0,\ y_0) = 0$ e $\dfrac{\partial f}{\partial y}\ (x_0,\ y_0) = 0$. Temos

$$\frac{\partial f}{\partial x}\ (x,\ y) = 2ax + by + d; \qquad \frac{\partial^2 f}{\partial x^2}\ (x,\ y) = 2a$$

$$\frac{\partial f}{\partial y}\ (x,\ y) = 2cy + bx + e; \qquad \frac{\partial^2 f}{\partial y^2}\ (x,\ y) = 2c \text{ e } \frac{\partial^2 f}{\partial x \partial y} = b.$$

Logo,

$f(x_0 + h, y_0 + k) - f(x_0, y_0) = \frac{1}{2}\left[2ah^2 + 2bhk + 2c\,k^2\right]$ e, portanto,

$f(x_0 + h, y_0 + k) - f(x_0, y_0) = ah^2 + bhk + ck^2.$

7. Do exercício ⑥ segue

$$f(x_0 + h, y_0 + k) - f(x_0, y_0) = a\,h^2 + b\,h\,k + c\,k^2$$

$$= a\left[h^2 + \frac{b}{a}\,h\,k + \frac{c}{a}\,k^2\right] = a\left[h^2 + \frac{b}{a}\,h\,k + \frac{b^2}{4a^2}\,k^2 - \frac{b^2}{4a^2}\,k^2 + \frac{c}{a}\,k^2\right]$$

$= a\left[\left(h + \frac{b}{2a}\,k\right)^2 + \frac{4ac - b^2}{4a^2}\,k^2\right] > 0$, para todo $(h, k) \neq (0, 0)$, pois $a > 0$ e $b^2 - 4ac < 0$.

Portanto, $f(x_0 + h, y_0 + h) > f(x_0, y_0)$, para todo $(h, k) \neq (0, 0)$. Logo (x_0, y_0) é um ponto de mínimo de f.

As curvas de nível de $f(x, y)$ são dadas pela equação
$ax^2 + b\,xy + cy^2 + dx + cy + m =$ constante.

Da Geometria Analítica sabemos que a equação representa uma elipse quando $b^2 - 4ac < 0$ e $a > 0$. Portanto, as curvas de nível são elipses e o gráfico de $f(x, y)$ é um parabolóide elíptico para cima.

Exercícios 15.5

1. ***a)*** $f(x, y) = x$ sen y e $(x_0, y_0) = (0, 0)$. O polinômio de Taylor de ordem 2 de f, em volta do ponto (0, 0) é dado por

$$P(x, y) = f(0, 0) + \frac{\partial f}{\partial x}(0, 0)x + \frac{\partial f}{\partial y}(0, 0)y$$
$$+ \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}(0, 0)x^2 + 2\,\frac{\partial^2 f}{\partial x \partial y}(0, 0)xy + \frac{\partial^2 f}{\partial y^2}(0, 0)y^2\right].$$

Temos $\frac{\partial f}{\partial x} = \text{sen }y;\ \frac{\partial f}{\partial x}(0, 0) = 0$

$$\frac{\partial f}{\partial y} = x\cos y;\ \frac{\partial f}{\partial y}(0, 0) = 0$$

$$\frac{\partial^2 f}{\partial x^2} = 0;\ \frac{\partial^2 f}{\partial y^2} = -x\,\text{sen }y;\ \frac{\partial^2 f}{\partial y^2} = (0, 0) = 0;\ \frac{\partial^2 f}{\partial x \partial y} = \cos y;\ \frac{\partial^2 f}{\partial x \partial y}(0, 0) = 1.$$

Portanto, $P(x, y) = xy.$

2. Seja $f(x, y) = x^3 + 2x^2y + 3y^3 + x - y$.

Tendo em vista que as derivadas parciais de ordens 4 são identicamente nulas, segue que $f(x, y)$ coincide com o seu polinômio de Taylor de 3.ª ordem. Então,

$$f(x, y) = f(1, 1) + \frac{\partial f}{\partial x}(1, 1)(x - 1) + \frac{\partial f}{\partial y}(1, 1)(y - 1) + \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}(1, 1)(x - 1)^2\right.$$
$$\left. + 2\frac{\partial^2 f}{\partial x \partial y}(1, 1)(x - 1)(y - 1) + \frac{\partial^2 f}{\partial y^2}(1, 1)(y - 1)^2\right] + \frac{1}{3!}\left[\frac{\partial^3 f}{\partial x^3}(1, 1)(x - 1)^3\right.$$
$$+ 3\frac{\partial^3 f}{\partial x^2 \partial y}(1, 1)(x - 1)^2(y - 1) + 3\frac{\partial^3 f}{\partial x \partial y^2}(1, 1)(x - 1)(y - 1)^2$$
$$\left. + \frac{\partial^3 f}{\partial y^3}(1, 1)(y - 1)^3\right].$$

Temos $f(1, 1) = 6$; $\frac{\partial f}{\partial x} = 3x^2 + 4x + 1$; $\frac{\partial f}{\partial x}(1, 1) = 8$; $\frac{\partial f}{\partial y} = 2x^2 + 9y^2 - 1$;

$\frac{\partial f}{\partial y}(1, 1) = 10$; $\frac{\partial^2 f}{\partial x^2} = 6x + 4$; $\frac{\partial^2 f}{\partial x^2}(1, 1) = 10$; $\frac{\partial^2 f}{\partial x \partial y} = 4x$; $\frac{\partial^2 f}{\partial x \partial y}(1, 1) = 4$;

$\frac{\partial^2 f}{\partial y^2} = 18y$; $\frac{\partial^2 f}{\partial y^2}(1, 1) = 18$; $\frac{\partial^3 f}{\partial x^3} = 6$; $\frac{\partial^3 f}{\partial x^2 \partial y} = 4$; $\frac{\partial^3 f}{\partial x \partial y^2} = 0$; $\frac{\partial^3 f}{\partial y^3} = 18$.

Portanto,

$$f(x, y) = 6 + 8(x - 1) + 10(y - 1) + 5(x - 1)^2 + 2(x - 1)(y - 1) + 9(y - 1)^2$$
$$+ (x - 1)^3 + 2(x - 1)^2(y - 1) + 3(y - 1)^3.$$