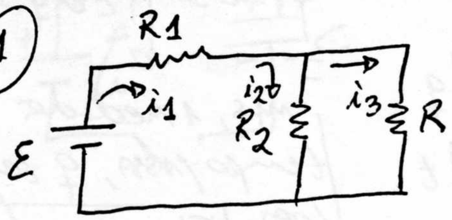
CAPÍTULO 10.

(1)



$R_1 = 20\Omega$

$R_2 = 60\Omega$

$i_1 = i_2 + i_3$

$$\boxed{i_2 = i_1 - i_3}$$

$$\begin{cases} \varepsilon - 20 i_1 - 60 i_2 = 0 \\ -60 i_2 + R i_3 = 0 \end{cases}$$

$$\boxed{i_2 = \frac{R i_3}{60}}$$

$$\varepsilon - 20(i_2 + i_3) - 60 i_2 = 0$$

$$\varepsilon - 80 i_2 - 20 i_3 = 0$$

$$\varepsilon - \frac{80 R i_3}{60} - 20 i_3 = 0$$

$$\varepsilon - \frac{4}{3} R i_3 - 20 i_3 = 0$$

$$i_3 = \left(20 + \frac{4}{3} R\right) = \varepsilon$$

$$\boxed{i_3 = \frac{\varepsilon}{\left(20 + \frac{4}{3} R\right)}}$$

A potência dissipada no resistor será:

$$P_R = R i_3^2 = \frac{R \varepsilon^2}{\left(20 + \frac{4}{3} R\right)^2}$$

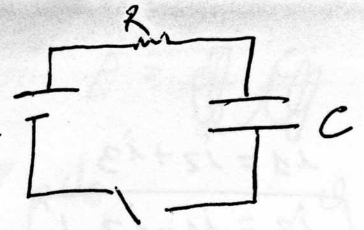
$$\frac{dP_R}{dR} = \frac{-\left(20 + \frac{4}{3} R\right)^2 \cdot \varepsilon^2 - R \varepsilon^2 \left[2\left(20 + 4/3\, R\right) \cdot 4/3\right]}{\left(20 + 4/3\, R\right)^2} = 0$$

$$\frac{dP_R}{dR} = \left(20 + \frac{4}{3} R\right)^{\cancel{2}} \cancel{\varepsilon^2} = R \cancel{\varepsilon^2} \frac{8}{3} \cancel{\left(20 + 4/3\, R\right)}$$

$$20 + \frac{4}{3} R = \frac{8}{3} R$$

$$20 = \frac{4}{3} R \Rightarrow R = \frac{60}{4} \Rightarrow \boxed{R = 15\Omega}$$

(2)



$$\varepsilon - Ri - \frac{q}{C} = 0$$

$$\begin{cases} dE_b = \varepsilon\, dq \\ dE_R = R i^2 dt \end{cases}$$

$$E_b = \int_0^q \varepsilon\, dq = \varepsilon . q$$

MAS, A medida que o tempo passa, q tende a ε Logo, $\boxed{E_b = \varepsilon . \varepsilon C = C\varepsilon^2}$ que é A energia produzida pela bateria.

$$C = \frac{q}{V} \; ; \quad i(t) = i_0 e^{-t/\tau} \; ; \quad \tau = RC \; ; \quad \varepsilon = Ri$$

$$E_R = \int dE_R = \int_0^\infty R i^2 dt = R \int_0^\infty \left(\frac{\varepsilon}{R}\right)^2 e^{-2t/\tau} dt$$

$$\frac{\varepsilon^2}{R} \int_0^\infty e^{-2t/\tau} dt = \frac{\varepsilon^2}{R} \frac{-\tau}{2} \cdot e^{-2t/\tau} \Big|_0^\infty$$

$$e^{-2t/\tau}\Big|_0^\infty \qquad \frac{1}{e^\infty} - \frac{1}{e^0} \quad (0-1)$$

$$\frac{\varepsilon^2}{R} \frac{\tau}{(-2)} (0-1) = \frac{\varepsilon^2}{\cancel{R}2} \cancel{R}C = \boxed{\frac{\varepsilon^2 C}{2}}$$

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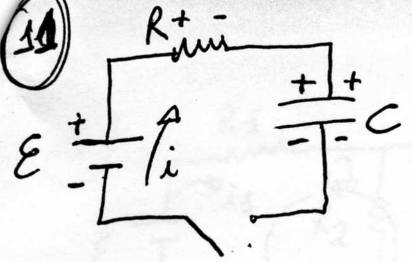

a) $V_\varepsilon + V_R + V_C = 0$

$$\begin{cases} V_\varepsilon = +\varepsilon \\ V_R = -R \cdot I \\ V_C = -\dfrac{Q}{C} \end{cases}$$

$$\varepsilon - R \cdot I - \frac{Q}{C} = 0 \quad \therefore \quad I = \frac{dQ}{dt}$$

$$-R\frac{dQ}{dt} - \frac{Q}{C} = -\varepsilon \Rightarrow R\frac{dQ}{dt} + \frac{Q}{C} = \varepsilon \quad \div (R)$$

$$\boxed{\frac{dQ}{dt} + \frac{Q}{RC} = \frac{\varepsilon}{R}}$$

$$+\varepsilon - R \cdot I - \frac{Q}{C} = 0$$

$$0 - R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0 \Rightarrow R\frac{dI}{dt} = -\frac{1}{C} I$$

$$\frac{dI}{I} = -\frac{dt}{RC} \qquad \tau_c = RC$$

$$\frac{dI}{I} = -\frac{dt}{\tau} \Rightarrow \int_{i_0}^{i} \frac{dI}{I} = -\int_0^t \frac{dt}{\tau_c}$$

$$\ln\frac{i}{i_0} = \frac{-t}{\tau_c}$$

$$\frac{i(t)}{i_0} = e^{-t/\tau_c} \Rightarrow \boxed{i(t) = i_0 \cdot e^{-t/\tau_c}}$$

$$\boxed{i(t) = \frac{\varepsilon}{R} \cdot e^{-t/RC}}$$