Chapter 3

Exercise Solutions

E3.1

$$-1 = 10\frac{\sin\alpha a}{\alpha a} + \cos\alpha a$$

By trial and error, $\alpha a = 5.305\ rad$

Now

$$\sqrt{\frac{2mE_2}{\hbar^2}}\cdot a = 5.305$$

so

$$E_2 = \frac{(5.305)^2\hbar^2}{2ma^2} = \frac{(5.305)^2\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)\left(5x10^{-10}\right)^2}$$

or

$$E_2 = 6.86x10^{-19}\ J = 4.29\ eV$$

Also

$$\sqrt{\frac{2mE_1}{\hbar^2}}\cdot a = \pi$$

so

$$E_1 = \frac{(\pi)^2(\hbar)^2}{2ma^2} = \frac{(\pi)^2\left(1.054x10^{-34}\right)^2}{2\left(9.11x10^{-31}\right)\left(5x10^{-10}\right)^2}$$

or

$$E_1 = 2.41x10^{-19}\ J = 1.50\ eV$$

Then

$$\Delta E = E_2 - E_1 = 4.29 - 1.50$$

or

$$\underline{\Delta E = 2.79\ eV}$$

E3.2

$$g_c(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_c}$$

Then

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\int_{E_c}^{E_c+kT}\left(E - E_C\right)^{1/2}dE$$

$$= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)\left(E - E_c\right)^{3/2}\Bigg|_{E_c}^{E_c+kT}$$

or

$$g_T = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

which yields

$$g_T = \frac{4\pi\left[2(1.08)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

which yields

$$\underline{g_T = 2.12x10^{25}\ m^{-3} = 2.12x10^{19}\ cm^{-3}}$$

E3.3

We have

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_v-kT}^{E_v}\left(E_v - E\right)^{1/2}dE$$

which yields

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_v - E\right)^{3/2}\Bigg|_{E_v-kT}^{E_v}$$

or

$$g_T = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[0 - (kT)^{3/2}\right]$$

$$= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{2}{3}\right)(kT)^{3/2}$$

Then

$$g_T = \frac{4\pi\left[2(0.56)\left(9.11x10^{-31}\right)\right]^{3/2}}{\left(6.625x10^{-34}\right)^3}\left(\frac{2}{3}\right)$$

$$\times\left[(0.0259)\left(1.6x10^{-19}\right)\right]^{3/2}$$

or

$$\underline{g_T = 7.92x10^{24}\,m^{-3} = 7.92x10^{18}\ cm^{-3}}$$

E3.4

(a)

$$f_F = \frac{1}{1+\exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_c - E_F}{kT}\right)}$$

or

$$f_F = \frac{1}{1+\exp\left(\frac{0.30}{0.0259}\right)} \Rightarrow \underline{f_F = 9.32x10^{-6}}$$

(b)

$$f_F = \frac{1}{1+\exp\left(\frac{0.30+0.0259}{0.0259}\right)} \Rightarrow$$

$$\underline{f_F = 3.43x10^{-6}}$$

E3.5

(a)

$$1-f_F = 1-\frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

$$= \frac{\exp\left(\frac{E-E_F}{kT}\right)}{1+\exp\left(\frac{E-E_F}{kT}\right)} = \frac{1}{1+\exp\left(\frac{E_F-E}{kT}\right)}$$

Then

$$1-f_F = \frac{1}{1+\exp\left(\frac{0.35}{0.0259}\right)}$$

so

$$\underline{1-f_F = 1.35x10^{-6}}$$

(b)

$$1-f_F = \frac{1}{1+\exp\left(\frac{0.35+0.0259}{0.0259}\right)}$$

or

$$\underline{1-f_F = 4.98x10^{-7}}$$

E3.6

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

(a)

$$f_F = \frac{1}{1+\exp\left(\frac{0.30}{0.03453}\right)} \Rightarrow \underline{f_F = 1.69x10^{-4}}$$

(b)

$$f_F = \frac{1}{1+\exp\left(\frac{0.30+0.03453}{0.03453}\right)} \Rightarrow$$

$$\underline{f_F = 6.20x10^{-5}}$$

E3.7

$$kT = 0.03453\ eV$$

(a)

$$1-f_F = \frac{1}{1+\exp\left(\frac{0.35}{0.03453}\right)} \Rightarrow$$

$$\underline{1-f_F = 3.96x10^{-5}}$$

(b)

$$1-f_F = \frac{1}{1+\exp\left(\frac{0.35+0.03453}{0.03453}\right)} \Rightarrow$$

$$\underline{1-f_F = 1.46x10^{-5}}$$
