Chapter 1

Problem Solutions

1.1

(a) fcc: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
Total of 4 atoms per unit cell

(b) bcc: 8 corner atoms $\times$ 1/8 = 1 atom
1 enclosed atom = 1 atom
Total of 2 atoms per unit cell

(c) Diamond: 8 corner atoms $\times$ 1/8 = 1 atom
6 face atoms $\times$ ½ = 3 atoms
4 enclosed atoms = 4 atoms
Total of 8 atoms per unit cell

1.2

(a) 4 Ga atoms per unit cell

$$\text{Density} = \frac{4}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ga} = 2.22x10^{22}\ cm^{-3}$$

4 As atoms per unit cell, so that

$$\text{Density of As} = 2.22x10^{22}\ cm^{-3}$$

(b)
8 Ge atoms per unit cell

$$\text{Density} = \frac{8}{\left(5.65x10^{-8}\right)^3} \Rightarrow$$

$$\text{Density of Ge} = 4.44x10^{22}\ cm^{-3}$$

1.3

(a) Simple cubic lattice; $a = 2r$

$$\text{Unit cell vol} = a^3 = (2r)^3 = 8r^3$$

$$\text{1 atom per cell, so atom vol.} = (1)\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{\left(\frac{4\pi r^3}{3}\right)}{8r^3} \times 100\% \Rightarrow \quad Ratio = 52.4\%$$

(b) Face-centered cubic lattice

$$d = 4r = a\sqrt{2} \Rightarrow \quad a = \frac{d}{\sqrt{2}} = 2\sqrt{2}\,r$$

$$\text{Unit cell vol} = a^3 = \left(2\sqrt{2}\,r\right)^3 = 16\sqrt{2}\,r^3$$

$$\text{4 atoms per cell, so atom vol.} = 4\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{4\left(\frac{4\pi r^3}{3}\right)}{16\sqrt{2}\,r^3} \times 100\% \Rightarrow \quad Ratio = 74\%$$

(c) Body-centered cubic lattice

$$d = 4r = a\sqrt{3} \Rightarrow a = \frac{4}{\sqrt{3}}r$$

$$\text{Unit cell vol.} = a^3 = \left(\frac{4}{\sqrt{3}}r\right)^3$$

$$\text{2 atoms per cell, so atom vol.} = 2\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{2\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{4r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 68\%$$

(d) Diamond lattice

$$\text{Body diagonal} = d = 8r = a\sqrt{3} \Rightarrow a = \frac{8}{\sqrt{3}}r$$

$$\text{Unit cell vol.} = a^3 = \left(\frac{8r}{\sqrt{3}}\right)^3$$

$$\text{8 atoms per cell, so atom vol. } 8\left(\frac{4\pi r^3}{3}\right)$$

Then

$$Ratio = \frac{8\left(\frac{4\pi r^3}{3}\right)}{\left(\frac{8r}{\sqrt{3}}\right)^3} \times 100\% \Rightarrow \quad Ratio = 34\%$$

1.4

From Problem 1.3, percent volume of fcc atoms is 74%; Therefore after coffee is ground,

$$\text{Volume} = 0.74\ cm^3$$

1.5

(a) $a = 5.43\ \mathring{A}$ From 1.3d, $a = \frac{8}{\sqrt{3}}r$

so that $r = \frac{a\sqrt{3}}{8} = \frac{(5.43)\sqrt{3}}{8} = 1.18\ \mathring{A}$

Center of one silicon atom to center of nearest neighbor $= 2r \Rightarrow \underline{2.36\ \mathring{A}}$

(b) Number density

$= \frac{8}{(5.43x10^{-8})^3} \Rightarrow$ Density $= 5x10^{22}\ cm^{-3}$

(c) Mass density

$= \rho = \frac{N(At.Wt.)}{N_A} = \frac{(5x10^{22})(28.09)}{6.02x10^{23}} \Rightarrow$

$\underline{\rho = 2.33\ grams / cm^3}$

1.6

(a) $a = 2r_A = 2(1.02) = 2.04\ \mathring{A}$

Now

$2r_A + 2r_B = a\sqrt{3} \Rightarrow 2r_B = 2.04\sqrt{3} - 2.04$

so that $r_B = 0.747\ \mathring{A}$

(b) A-type; 1 atom per unit cell

Density $= \frac{1}{(2.04x10^{-8})^3} \Rightarrow$

Density(A) $= \underline{1.18x10^{23}\ cm^{-3}}$

B-type: 1 atom per unit cell, so

Density(B) $= \underline{1.18x10^{23}\ cm^{-3}}$

1.7

(b)

$a = 1.8 + 1.0 \Rightarrow \underline{a = 2.8\ \mathring{A}}$

(c)

Na: Density $= \frac{1/2}{(2.8x10^{-8})^3}\ \underline{= 2.28x10^{22}\ cm^{-3}}$

Cl: Density (same as Na) $\underline{= 2.28x10^{22}\ cm^{-3}}$

(d)

Na: At.Wt. = 22.99

Cl: At. Wt. = 35.45

So, mass per unit cell

$= \frac{\frac{1}{2}(22.99) + \frac{1}{2}(35.45)}{6.02x10^{23}} = 4.85x10^{-23}$

Then mass density is

$\rho = \frac{4.85x10^{-23}}{(2.8x10^{-8})^3} \Rightarrow$

$\underline{\rho = 2.21\ gm / cm^3}$

1.8

(a) $a\sqrt{3} = 2(2.2) + 2(1.8) = 8\ \mathring{A}$

so that

$\underline{a = 4.62\ \mathring{A}}$

Density of A $= \frac{1}{(4.62x10^{-8})^3} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$

Density of B $= \frac{1}{(4.62x10^{-8})} \Rightarrow \underline{1.01x10^{22}\ cm^{-3}}$

(b) Same as (a)

(c) Same material

1.9

(a) Surface density

$= \frac{1}{a^2\sqrt{2}} = \frac{1}{(4.62x10^{-8})^2\sqrt{2}} \Rightarrow$

$\underline{3.31x10^{14}\ cm^{-2}}$

Same for A atoms and B atoms

(b) Same as (a)

(c) Same material

1.10

(a) Vol density $= \frac{1}{a_o^3}$

Surface density $= \frac{1}{a_o^2\sqrt{2}}$

(b) Same as (a)

1.11

Sketch

1.12

(a)

$\left(\frac{1}{1}, \frac{1}{3}, \frac{1}{1}\right) \Rightarrow \underline{(313)}$

(b)

$\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right) \Rightarrow \underline{(121)}$

1.13

(a) Distance between nearest (100) planes is:

$$\underline{d = a = 5.63\ \mathring{A}}$$

(b) Distance between nearest (110) planes is:

$$d = \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{5.63}{\sqrt{2}}$$

or

$$\underline{d = 3.98\ \mathring{A}}$$

(c) Distance between nearest (111) planes is:

$$d = \frac{1}{3}a\sqrt{3} = \frac{a}{\sqrt{3}} = \frac{5.63}{\sqrt{3}}$$

or

$$\underline{d = 3.25\ \mathring{A}}$$

1.14

(a)

Simple cubic: $a = 4.50\ \mathring{A}$

(i) (100) plane, surface density,

$$= \frac{1\ atom}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{4.94x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{1\ atom}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{3.49x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{3\left(\frac{1}{6}\right)atoms}{\frac{1}{2}\left(a\sqrt{2}\right)(x)} = \frac{\frac{1}{2}}{\frac{1}{2}\cdot a\sqrt{2}\cdot\frac{a\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}a^2}$$

$$= \frac{1}{\sqrt{3}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{2.85x10^{14}\ cm^{-2}}$$

(b)

Body-centered cubic

(i) (100) plane, surface density,

Same as (a),(i); surface density $\underline{4.94x10^{14}\ cm^{-2}}$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

Same as (a),(iii), surface density $\underline{2.85x10^{14}\ cm^{-2}}$

(c)

Face centered cubic

(i) (100) plane, surface density

$$= \frac{2\ atoms}{\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{9.88x10^{14}\ cm^{-2}}$$

(ii) (110) plane, surface density,

$$= \frac{2\ atoms}{\sqrt{2}\left(4.50x10^{-8}\right)^2} \Rightarrow \underline{6.99x10^{14}\ cm^{-2}}$$

(iii) (111) plane, surface density,

$$= \frac{\left(3\cdot\frac{1}{6}+3\cdot\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}a^2} = \frac{4}{\sqrt{3}\left(4.50x10^{-8}\right)^2}$$

or $\underline{1.14x10^{15}\ cm^{-2}}$

1.15

(a)

(100) plane of silicon – similar to a fcc,

$$\text{surface density} = \frac{2\ atoms}{\left(5.43x10^{-8}\right)^2} \Rightarrow$$

$$\underline{6.78x10^{14}\ cm^{-2}}$$

(b)

(110) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{2}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{9.59x10^{14}\ cm^{-2}}$$

(c)

(111) plane, surface density,

$$= \frac{4\ atoms}{\sqrt{3}\left(5.43x10^{-8}\right)^2} \Rightarrow \underline{7.83x10^{14}\ cm^{-2}}$$

1.16

$$d = 4r = a\sqrt{2}$$

then

$$a = \frac{4r}{\sqrt{2}} = \frac{4(2.25)}{\sqrt{2}} = 6.364\ \mathring{A}$$

(a)

$$\text{Volume Density} = \frac{4\ atoms}{\left(6.364x10^{-8}\right)^3} \Rightarrow$$

$$\underline{1.55x10^{22}\ cm^{-3}}$$

(b)

Distance between (110) planes,

$$= \frac{1}{2}a\sqrt{2} = \frac{a}{\sqrt{2}} = \frac{6.364}{\sqrt{2}} \Rightarrow$$

or

$\underline{4.50\ \overset{\circ}{A}}$

(c)
Surface density

$$= \frac{2\ atoms}{\sqrt{2}a^2} = \frac{2}{\sqrt{2}\left(6.364x10^{-8}\right)^2}$$

or

$\underline{3.49x10^{14}\ cm^{-2}}$

1.17

Density of silicon atoms $= 5x10^{22}\ cm^{-3}$ and 4 valence electrons per atom, so
Density of valence electrons $\underline{2x10^{23}\ cm^{-3}}$

1.18

Density of GaAs atoms

$$= \frac{8\ atoms}{\left(5.65x10^{-8}\right)^3} = 4.44x10^{22}\ cm^{-3}$$

An average of 4 valence electrons per atom,
Density of valence electrons $\underline{1.77x10^{23}\ cm^{-3}}$

1.19

(a) Percentage $= \dfrac{2x10^{16}}{5x10^{22}} x100\% \Rightarrow$

$\underline{4x10^{-5}\%}$

(b) Percentage $= \dfrac{1x10^{15}}{5x10^{22}} x100\% \Rightarrow$

$\underline{2x10^{-6}\%}$

1.20

(a) Fraction by weight $\approx \dfrac{\left(5x10^{16}\right)(30.98)}{\left(5x10^{22}\right)(28.06)} \Rightarrow$

$\underline{1.10x10^{-6}}$

(b) Fraction by weight

$$\approx \frac{\left(10^{18}\right)(10.82)}{\left(5x10^{16}\right)(30.98) + \left(5x10^{22}\right)(28.06)} \Rightarrow$$

$\underline{7.71x10^{-6}}$

1.21

Volume density $= \dfrac{1}{d^3} = 2x10^{15}\ cm^{-3}$

So

$$d = 7.94x10^{-6}\ cm = 794\ \overset{\circ}{A}$$

We have $a_o = 5.43\ \overset{\circ}{A}$

So

$$\frac{d}{a_o} = \frac{794}{5.43} \Rightarrow \underline{\frac{d}{a_o} = 146}$$