Chapter 2

Problem Solutions

2.1 Computer plot

2.2 Computer plot

2.3 Computer plot

2.4

For problem 2.2; Phase $= \frac{2\pi x}{\lambda} - \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = +\omega\left(\frac{\lambda}{2\pi}\right)}$$

For problem 2.3; Phase $= \frac{2\pi x}{\lambda} + \omega t =$ constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0 \text{ or } \underline{\frac{dx}{dt} = v_p = -\omega\left(\frac{\lambda}{2\pi}\right)}$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90\ eV = (4.90)(1.6x10^{-19})\ J$

So

$$\lambda = \frac{(6.625x10^{-34})(3x10^{10})}{(4.90)(1.6x10^{-19})} \Rightarrow 2.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.254\ \mu m}$$

Cesium: $E = 1.90\ eV = (1.90)(1.6x10^{-19})\ J$

So

$$\lambda = \frac{(6.625x10^{-34})(3x10^{10})}{(1.90)(1.6x10^{-19})} \Rightarrow 6.54x10^{-5}\ cm$$

or

$$\underline{\lambda = 0.654\ \mu m}$$

2.6

(a) Electron: (i) K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2(9.11x10^{-31})(1.6x10^{-19})}$$

or

$$\underline{p = 5.4x10^{-25}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{5.4x10^{-25}} \Rightarrow$$

or

$$\underline{\lambda = 12.3\ \overset{\circ}{A}}$$

(ii) K.E. $= T = 100\ eV = 1.6x10^{-17}\ J$

$$p = \sqrt{2mT} \Rightarrow \underline{p = 5.4x10^{-24}\ kg - m/s}$$

$$\lambda = \frac{h}{p} \Rightarrow \underline{\lambda = 1.23\ \overset{\circ}{A}}$$

(b) Proton: K.E. $= T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT} = \sqrt{2(1.67x10^{-27})(1.6x10^{-19})}$$

or

$$\underline{p = 2.31x10^{-23}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{2.31x10^{-23}} \Rightarrow$$

or

$$\underline{\lambda = 0.287\ \overset{\circ}{A}}$$

(c) Tungsten Atom: At. Wt. = 183.92

For $T = 1\ eV = 1.6x10^{-19}\ J$

$$p = \sqrt{2mT}$$
$$= \sqrt{2(183.92)(1.66x10^{-27})(1.6x10^{-19})}$$

or

$$\underline{p = 3.13x10^{-22}\ kg = m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{3.13x10^{-22}} \Rightarrow$$

or

$$\underline{\lambda = 0.0212\ \overset{\circ}{A}}$$

(d) A 2000 kg traveling at 20 m/s:

$$p = mv = (2000)(20) \Rightarrow$$

or

$$\underline{p = 4x10^{4}\ kg - m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{4x10^{4}} \Rightarrow$$

or

$$\underline{\lambda = 1.66x10^{-28}\ \overset{\circ}{A}}$$

2.7

$$E_{avg} = \frac{3}{2}kT = \frac{3}{2}(0.0259) \Rightarrow$$

or

$$\underline{E_{avg} = 0.01727\ eV}$$

Now

$$p_{avg} = \sqrt{2mE_{avg}}$$

$$= \sqrt{2(9.11x10^{-31})(0.01727)(1.6x10^{-19})}$$

or

$$\underline{p_{avg} = 7.1x10^{-26}\ kg - m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{7.1x10^{-26}} \Rightarrow$$

or

$$\underline{\lambda = 93.3\ \overset{\circ}{A}}$$

2.8

$$E_p = h\nu_p = \frac{hc}{\lambda_p}$$

Now

$$E_e = \frac{p_e^2}{2m} \text{ and } p_e = \frac{h}{\lambda_e} \Rightarrow E_e = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p = E_e$ and $\lambda_p = 10\lambda_e$

Then

$$\frac{hc}{\lambda_p} = \frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2 = \frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p = \frac{100h}{2mc}$$

$$E_p = E = \frac{hc}{\lambda_p} = \frac{hc}{100h}\cdot 2mc = \frac{2mc^2}{100}$$

$$= \frac{2(9.11x10^{-31})(3x10^8)^2}{100} \Rightarrow$$

So

$$\underline{E = 1.64x10^{-15}\ J = 10.3\ keV}$$

2.9

(a) $E = \frac{1}{2}mv^2 = \frac{1}{2}(9.11x10^{-31})(2x10^4)^2$

or

$$E = 1.822x10^{-22}\ J \Rightarrow \underline{E = 1.14x10^{-3}\ eV}$$

Also

$$p = mv = (9.11x10^{-31})(2x10^4) \Rightarrow$$

$$\underline{p = 1.822x10^{-26}\ kg - m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{1.822x10^{-26}} \Rightarrow$$

$$\underline{\lambda = 364\ \overset{\circ}{A}}$$

(b)

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{125x10^{-10}} \Rightarrow$$

$$\underline{p = 5.3x10^{-26}\ kg - m/s}$$

Also

$$v = \frac{p}{m} = \frac{5.3x10^{-26}}{9.11x10^{-31}} = 5.82x10^4\ m/s$$

or

$$\underline{v = 5.82x10^6\ cm/s}$$

Now

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(9.11x10^{-31})(5.82x10^4)^2$$

or

$$E = 1.54x10^{-21}\ J \Rightarrow \underline{E = 9.64x10^{-3}\ eV}$$

2.10

(a) $E = h\nu = \frac{hc}{\lambda} = \frac{(6.625x10^{-34})(3x10^8)}{1x10^{-10}}$

or

$$E = 1.99x10^{-15}\ J$$

Now

$$E = e\cdot V \Rightarrow 1.99x10^{-15} = (1.6x10^{-19})V$$

so

$$\underline{V = 12.4x10^3\ V = 12.4\ kV}$$

(b) $p = \sqrt{2mE} = \sqrt{2(9.11x10^{-31})(1.99x10^{-15})}$

$$= 6.02x10^{-23}\ kg - m/s$$

Then

$$\lambda = \frac{h}{p} = \frac{6.625x10^{-34}}{6.02x10^{-23}} \Rightarrow \underline{\lambda = 0.11\ \overset{\circ}{A}}$$

2.11

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-6}} \Rightarrow$

$\underline{\Delta p = 1.054x10^{-28}\ kg - m/s}$

(b)

$$E = \frac{hc}{\lambda} = hc\left(\frac{p}{h}\right) = pc$$

So

$$\Delta E = c(\Delta p) = \left(3x10^{8}\right)\left(1.054x10^{-28}\right) \Rightarrow$$

or

$\Delta E = 3.16x10^{-20}\ J \Rightarrow \underline{\Delta E = 0.198\ eV}$

2.12

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{12x10^{-10}} \Rightarrow$

$\underline{\Delta p = 8.78x10^{-26}\ kg - m/s}$

(b)

$$\Delta E = \frac{1}{2}\cdot\frac{(\Delta p)^2}{m} = \frac{1}{2}\cdot\frac{\left(8.78x10^{-26}\right)^2}{5x10^{-29}} \Rightarrow$$

$\Delta E = 7.71x10^{-23}\ J \Rightarrow \underline{\Delta E = 4.82x10^{-4}\ eV}$

2.13

(a) Same as 2.12 (a), $\underline{\Delta p = 8.78x10^{-26}\ kg - m/s}$

(b)

$$\Delta E = \frac{1}{2}\cdot\frac{(\Delta p)^2}{m} = \frac{1}{2}\cdot\frac{\left(8.78x10^{-26}\right)^2}{5x10^{-26}} \Rightarrow$$

$\Delta E = 7.71x10^{-26}\ J \Rightarrow \Delta E = 4.82x10^{-7}\ eV$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-2}} = 1.054x10^{-32}$$

$$p = mv \Rightarrow \Delta v = \frac{\Delta p}{m} = \frac{1.054x10^{-32}}{1500} \Rightarrow$$

or

$\underline{\Delta v = 7x10^{-36}\ m/s}$

2.15

(a) $\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054x10^{-34}}{10^{-10}} \Rightarrow$

$\underline{\Delta p = 1.054x10^{-24}\ kg - m/s}$

(b) $\Delta t = \frac{1.054x10^{-34}}{(1)\left(1.6x10^{-19}\right)} \Rightarrow$

or

$\underline{\Delta t = 6.6x10^{-16}\ s}$

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$+V(x)\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t) + \Psi_2(x,t)$ is also a solution.

(b)

If $\Psi_1 \cdot \Psi_2$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(\Psi_1\cdot\Psi_2\right) + V(x)\left(\Psi_1\cdot\Psi_2\right)$$
$$= j\hbar\frac{\partial}{\partial t}\left(\Psi_1\cdot\Psi_2\right)$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1\frac{\partial^2\Psi_2}{\partial x^2} + \Psi_2\frac{\partial^2\Psi_1}{\partial x^2} + 2\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x)\Psi_1\cdot\Psi_2 = j\hbar\left[\Psi_1\frac{\partial\Psi_2}{\partial t} + \Psi_2\frac{\partial\Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1 \cdot \Psi_2$ we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + \frac{1}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial\Psi_1}{\partial x}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we are left with

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we may write

$$\frac{-\hbar^2}{2m}\frac{1}{\Psi_2}\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\Psi(x,t)=A[\sin(\pi x)]\exp(-j\omega t)$$

$$\int_{-1}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{-1}^{+1}\sin^2(\pi x)dx$$

or

$$|A^2|\cdot\left[\frac{1}{2}x-\frac{1}{4\pi}\sin(2\pi x)\right]_{-1}^{+1}=1$$

which yields

$$\underline{|A^2|=1} \text{ or } \underline{A=+1,-1,+j,-j}$$

2.18

$$\Psi(x,t)=A[\sin(n\pi x)]\exp(-j\omega t)$$

$$\int_{0}^{+1}|\Psi(x,t)|^2dx=1=|A|^2\int_{0}^{+1}\sin^2(n\pi x)dx$$

or

$$|A|^2\cdot\left[\frac{1}{2}x-\frac{1}{4n\pi}\sin(2n\pi x)\right]_{0}^{+1}=1$$

which yields

$$\underline{|A|^2=2} \text{ or}$$

$$\underline{A=+\sqrt{2},-\sqrt{2},+j\sqrt{2},-j\sqrt{2}}$$

2.19

Note that $\int_0^\infty \Psi\cdot\Psi^* dx=1$

Function has been normalized

(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P=-1\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields

$$\underline{P=0.393}$$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P=-1\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields

$$\underline{P=0.239}$$

(c)

$$P=\int_0^{a_o}\left(\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right)^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

or

$$P=-1[\exp(-2)-1]$$

which yields

$$\underline{P=0.865}$$

2.20

(a) $kx - \omega t = \text{constant}$

Then

$$k\frac{dx}{dt} - \omega = 0 \Rightarrow \frac{dx}{dt} = v_p = +\frac{\omega}{k}$$

or

$$v_p = \frac{1.5x10^{13}}{1.5x10^{9}} = 10^4 \ m/s$$

$$\underline{v_p = 10^6 \ cm/s}$$

(b)

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5x10^9}$$

or

$$\underline{\lambda = 41.9 \ \mathring{A}}$$

Also

$$p = \frac{h}{\lambda} = \frac{6.625x10^{-34}}{41.9x10^{-10}} \Rightarrow$$

or

$$\underline{p = 1.58x10^{-25} \ kg - m/s}$$

Now

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{41.9x10^{-10}}$$

or

$$E = 4.74x10^{-17} \ J \Rightarrow \underline{E = 2.96x10^2 \ eV}$$

2.21

$$\underline{\psi(x) = A\exp\left[-j\left(kx + \omega t\right)\right]}$$

where

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$= \frac{\sqrt{2\left(9.11x10^{-31}\right)\left(0.015\right)\left(1.6x10^{-19}\right)}}{1.054x10^{-34}}$$

or

$$\underline{k = 6.27x10^8 \ m^{-1}}$$

Now

$$\omega = \frac{E}{\hbar} = \frac{\left(0.015\right)\left(1.6x10^{-19}\right)}{1.054x10^{-34}}$$

or

$$\underline{\omega = 2.28x10^{13} \ rad/s}$$

2.22

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{\left(1.054x10^{-34}\right)^2 \pi^2 n^2}{2\left(9.11x10^{-31}\right)\left(100x10^{-10}\right)^2}$$

so

$$E = 6.018x10^{-22} n^2 \ (J)$$

or

$$E = 3.76x10^{-3} n^2 \ (eV)$$

Then

$$n = 1 \Rightarrow \underline{E_1 = 3.76x10^{-3} \ eV}$$

$$n = 2 \Rightarrow \underline{E_2 = 1.50x10^{-2} \ eV}$$

$$n = 3 \Rightarrow \underline{E_3 = 3.38x10^{-2} \ eV}$$

2.23

(a) $E = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$= \frac{\left(1.054x10^{-34}\right)^2 \pi^2 n^2}{2\left(9.11x10^{-31}\right)\left(12x10^{-10}\right)^2}$$

$$= 4.81x10^{-20} n^2 \ (J)$$

So

$$E_1 = 4.18x10^{-20} \ J \Rightarrow \underline{E_1 = 0.261 \ eV}$$

$$E_2 = 1.67x10^{-19} \ J \Rightarrow \underline{E_2 = 1.04 \ eV}$$

(b)

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$$

or

$$\lambda = \frac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{1.67x10^{-19} - 4.18x10^{-20}} \Rightarrow$$

$$\lambda = 1.59x10^{-6} \ m$$

or

$$\underline{\lambda = 1.59 \ \mu m}$$

2.24

(a) For the infinite potential well

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} \Rightarrow n^2 = \frac{2ma^2 E}{\hbar^2 \pi^2}$$

so

$$n^2 = \frac{2\left(10^{-5}\right)\left(10^{-2}\right)^2\left(10^{-2}\right)}{\left(1.054x10^{-34}\right)^2 \pi^2} = 1.82x10^{56}$$

or

$$\underline{n = 1.35x10^{28}}$$

(b)

$$\Delta E = \frac{\hbar^2\pi^2}{2ma^2}\left[(n+1)^2 - n^2\right]$$

$$= \frac{\hbar^2\pi^2}{2ma^2}(2n+1)$$

or

$$\Delta E = \frac{\left(1.054x10^{-34}\right)^2 \pi^2 (2)\left(1.35x10^{28}\right)}{2\left(10^{-5}\right)\left(10^{-2}\right)^2}$$

$$\underline{\Delta E = 1.48x10^{-30}\ J}$$

Energy in the (n+1) state is $1.48x10^{-30}$ Joules larger than 10 mJ.

(c)

Quantum effects would not be observable.

2.25

For a neutron and n = 1:

$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{\left(1.054x10^{-34}\right)\pi^2}{2\left(1.66x10^{-27}\right)\left(10^{-14}\right)^2}$$

or

$$\underline{E_1 = 2.06x10^6\ eV}$$

For an electron in the same potential well:

$$E_1 = \frac{\left(1.054x10^{-34}\right)^2\pi^2}{2\left(9.11x10^{-31}\right)\left(10^{-14}\right)^2}$$

or

$$\underline{E_1 = 3.76x10^9\ eV}$$

2.26

Schrodinger's wave equation

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$$

We know that

$$\psi(x) = 0 \text{ for } x \ge \frac{a}{2} \text{ and } x \le \frac{-a}{2}$$

$$V(x) = 0 \text{ for } \frac{-a}{2} \le x \le \frac{+a}{2}$$

so in this region

$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$

Solution is of the form

$$\psi(x) = A\cos Kx + B\sin Kx$$

where $K = \sqrt{\dfrac{2mE}{\hbar^2}}$

Boundary conditions:

$$\psi(x) = 0 \text{ at } x = \frac{+a}{2},\ x = \frac{-a}{2}$$

So, first mode:

$$\underline{\psi_1(x) = A\cos Kx}$$

where $\underline{K = \dfrac{\pi}{a}}$ so $\underline{E_1 = \dfrac{\pi^2\hbar^2}{2ma^2}}$

Second mode:

$$\underline{\psi_2(x) = B\sin Kx}$$

where $\underline{K = \dfrac{2\pi}{a}}$ so $\underline{E_2 = \dfrac{4\pi^2\hbar^2}{2ma^2}}$

Third mode:

$$\underline{\psi_3(x) = A\cos Kx}$$

where $\underline{K = \dfrac{3\pi}{a}}$ so $\underline{E_3 = \dfrac{9\pi^2\hbar^2}{2ma^2}}$

Fourth mode:

$$\underline{\psi_4(x) = B\sin Kx}$$

where $\underline{K = \dfrac{4\pi}{a}}$ so $\underline{E_4 = \dfrac{16\pi^2\hbar^2}{2ma^2}}$

2.27

The 3-D wave equation in cartesian coordinates, for V(x,y,z) = 0

$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$

Use separation of variables, so let

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Substituting into the wave equation, we get

$$YZ\frac{\partial^2 X}{\partial x^2} + XZ\frac{\partial^2 Y}{\partial y^2} + XY\frac{\partial^2 Z}{\partial z^2} + \frac{2mE}{\hbar^2}XYZ = 0$$

Dividing by XYZ and letting $k^2 = \dfrac{2mE}{\hbar^2}$, we obtain

$$(1)\quad \frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

We may set

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2 \text{ so } \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form

$$X(x) = A\sin(k_x x) + B\cos(k_x x)$$

Boundary conditions: $X(0) = 0 \Rightarrow B = 0$

and $X(x=a) = 0 \Rightarrow k_x = \dfrac{n_x \pi}{a}$

where $n_x = 1, 2, 3, \ldots$

Similarly, let

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2 \text{ and } \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2} = -k_z^2$$

Applying the boundary conditions, we find

$$k_y = \frac{n_y \pi}{a},\ n_y = 1, 2, 3, \ldots$$

$$k_z = \frac{n_z \pi}{a},\ n_z = 1, 2, 3, \ldots$$

From Equation (1) above, we have

$$-k_x^2 - k_y^2 - k_z^2 + k^2 = 0$$

or

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \frac{2mE}{\hbar^2}$$

so that

$$E \Rightarrow E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

2.28

For the 2-dimensional infinite potential well:

$$\frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} + \frac{2mE}{\hbar^2}\psi(x,y) = 0$$

Let $\psi(x,y) = X(x)Y(y)$

Then substituting,

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2}XY = 0$$

Divide by XY

So

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} + \frac{2mE}{\hbar^2} = 0$$

Let

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2} = -k_x^2$$

or

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0$$

Solution is of the form:

$$X = A\sin(k_x x) + B\cos(k_x x)$$

But $X(x=0) = 0 \Rightarrow B = 0$

So

$$X = A\sin(k_x x)$$

Also, $X(x=a) = 0 \Rightarrow k_x a = n_x \pi$

Where $n_x = 1, 2, 3, \ldots$

So that $k_x = \dfrac{n_x \pi}{a}$

We can also define

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

Solution is of the form

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

But

$$Y(y=0) = 0 \Rightarrow D = 0$$

and

$$Y(y=b) = 0 \Rightarrow k_y b = n_y \pi$$

so that

$$k_y = \frac{n_y \pi}{b}$$

Now

$$-k_x^2 - k_y^2 + \frac{2mE}{\hbar^2} = 0$$

which yields

$$E \Rightarrow E_{n_x n_y} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

Similarities: energy is quantized
Difference: now a function of 2 integers

2.29

(a) Derivation of energy levels exactly the same as in the text.

(b) $\Delta E = \dfrac{\hbar^2 \pi^2}{2ma^2}\left(n_2^2 - n_1^2\right)$

For $n_2 = 2$, $n_1 = 1$

Then

$$\Delta E = \frac{3\hbar^2 \pi^2}{2ma^2}$$

(i) $a = 4\ \overset{\circ}{A}$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(4x10^{-10}\right)^2} \Rightarrow$$

$$\Delta E = 3.85x10^{-3}\ eV$$

(ii) $a = 0.5\ cm$

$$\Delta E = \frac{3\left(1.054x10^{-34}\right)^2 \pi^2}{2\left(1.67x10^{-27}\right)\left(0.5x10^{-2}\right)^2} \Rightarrow$$

$$\Delta E = 2.46x10^{-17}\ eV$$

2.30

(a) For region II, $x > 0$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

where

$$K_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave, and term with A_2 represents the reflected wave.

Region I, $x < 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

The general solution is of the form

$$\psi_1(x) = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

where

$$K_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave, and the term involving A_1 represents the reflected wave; but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1 \exp(-jK_1 x)$$

$$\psi_2(x) = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(b)

Boundary conditions:

(1) $\psi_1(x = 0) = \psi_2(x = 0)$

(2) $\left.\frac{\partial \psi_1(x)}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi_2(x)}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$K_2 A_2 - K_2 B_2 = -K_1 B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{K_2 - K_1}{K_2 + K_1}\right) B_2 \quad \text{and} \quad B_1 = \left(\frac{2K_2}{K_2 + K_1}\right) B_2$$

The reflection coefficient is

$$R = \frac{A_2 A_2^*}{B_2 B_2^*} \Rightarrow \quad R = \left(\frac{K_2 - K_1}{K_2 + K_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow \quad T = \frac{4K_1 K_2}{\left(K_1 + K_2\right)^2}$$

2.31

In region II, $x > 0$, we have

$$\psi_2(x) = A_2 \exp(-K_2 x)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

For $V_O = 2.4\ eV$ and $E = 2.1\ eV$

$$K_2 = \left\{\frac{2\left(9.11x10^{-31}\right)(2.4 - 2.1)\left(1.6x10^{-19}\right)}{\left(1.054x10^{-34}\right)^2}\right\}^{1/2}$$

or

$$K_2 = 2.81x10^9\ m^{-1}$$

Probability at x compared to x = 0, given by

$$P = \left|\frac{\psi_2(x)}{\psi_2(0)}\right|^2 = \exp(-2K_2 x)$$

(a) For $x = 12\ \overset{\circ}{A}$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(12x10^{-10}\right)\right] \Rightarrow$$

$$P = 1.18x10^{-3} = 0.118\,\%$$

(b) For $x = 48\ A°$

$$P = \exp\left[-2\left(2.81x10^9\right)\left(48x10^{-10}\right)\right] \Rightarrow$$

$$P = 1.9x10^{-10}\ \%$$

2.32

For $V_O = 6\ eV$, $E = 2.2\ eV$

We have that

$$T = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)$$

where

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(9.11x10^{-31})(6-2.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 9.98x10^9\ m^{-1}$$

For $a = 10^{-10}\ m$

$$T = 16\left(\frac{2.2}{6}\right)\left(1 - \frac{2.2}{6}\right)\exp\left[-2(9.98x10^9)(10^{-10})\right]$$

or

$$\underline{T = 0.50}$$

For $a = 10^{-9}\ m$

$$\underline{T = 7.97x10^{-9}}$$

2.33

Assume that Equation [2.62] is valid:

$$T = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)$$

(a) For $m = (0.067)m_o$

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 1.027x10^9\ m^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1 - \frac{0.2}{0.8}\right)\exp\left[-2(1.027x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 0.138}$$

(b) For $m = (1.08)m_o$

$$K_2 = \left\{\frac{2(1.08)(9.11x10^{-31})(0.8-0.2)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 4.124x10^9\ m^{-1}$$

Then

$$T = 3\exp\left[-2(4.124x10^9)(15x10^{-10})\right]$$

or

$$\underline{T = 1.27x10^{-5}}$$

2.34

$V_O = 10x10^6\ eV$, $E = 3x10^6\ eV$, $a = 10^{-14}\ m$

and $m = 1.67x10^{-27}\ kg$

Now

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(1.67x10^{-27})(10-3)(10^6)(1.6x10^{-19})}{(1.054x10^{-34})^2}\right\}^{1/2}$$

or

$$K_2 = 5.80x10^{14}\ m^{-1}$$

So

$$T = 16\left(\frac{3}{10}\right)\left(1 - \frac{3}{10}\right)\exp\left[-2(5.80x10^{14})(10^{-14})\right]$$

or

$$\underline{T = 3.06x10^{-5}}$$

2.35

Region I, $V = 0$ $(x < 0)$; Region II, $V = V_O$ $(0 < x < a)$; Region III, $V = 0$ $(x > a)$.

(a) Region I;

$$\psi_1(x) = A_1\exp(jK_1x) + B_1\exp(-jK_1x)$$

(incident) (reflected)

Region II;

$$\psi_2(x) = A_2\exp(K_2x) + B_2\exp(-K_2x)$$

Region III;

$$\psi_3(x) = A_3\exp(jK_1x) + B_3\exp(-jK_1x)$$

(b)

In region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into region III, there will not be a reflected wave which means that $B_3 = 0$.

(c)

Boundary conditions:

For $x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow jK_1A_1 - jK_1B_2 = K_2A_2 - K_2B_2$$

For $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2\exp(K_2a) + B_2\exp(-K_2a) = A_3\exp(jK_1a)$$

And also

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$K_2 A_2 \exp(K_2 a) - K_2 B_2 \exp(-K_2 a)$$
$$= jK_1 A_3 \exp(jK_1 a)$$

Transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4K_1K_2}\left\{\left(K_2^2 - K_1^2\right)\left[\exp(K_2 a) - \exp(-K_2 a)\right]\right.$$
$$\left.-2jK_1K_2\left[\exp(K_2 a) + \exp(-K_2 a)\right]\right\}\exp(jK_1 a)$$

We then find that

$$A_1 A_1^* = \frac{A_3 A_3^*}{\left(4K_1K_2\right)^2}\left\{\left(K_2^2 - K_1^2\right)\left[\exp(K_2 a)\right.\right.$$
$$\left.-\exp(-K_2 a)\right]^2$$
$$\left.+4K_1^2K_2^2\left[\exp(K_2 a) + \exp(-K_2 a)\right]^2\right\}$$

We have

$$K_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

and since $V_O >> E$, then $K_2 a$ will be large so that

$$\exp(K_2 a) >> \exp(-K_2 a)$$

Then we can write

$$A_1 A_1^* = \frac{A_3 A_3^*}{\left(4K_1K_2\right)^2}\left\{\left(K_2^2 - K_1^2\right)\left[\exp(K_2 a)\right]^2\right.$$
$$\left.+4K_1^2K_2^2\left[\exp(K_2 a)\right]^2\right\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{\left(4K_1K_2\right)^2}\left(K_2^2 + K_1^2\right)\exp(2K_2 a)$$

Substituting the expressions for K_1 and K_2, we find

$$K_1^2 + K_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$K_1^2K_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)(V_O = E)(E)$$

or

$$K_1^2K_2^2 = \left(\frac{2m}{\hbar^2}\right)^2 V_O\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^*\left(\dfrac{2mV_O}{\hbar^2}\right)^2 \exp(2K_2 a)}{16\left[\left(\dfrac{2m}{\hbar^2}\right)^2 V_O\left(1 - \dfrac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\dfrac{E}{V_O}\right)\left(1 - \dfrac{E}{V_O}\right)\exp(-2K_2 a)}$$

or finally

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2K_2 a)$$

2.36

Region I: $V = 0$

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1 = 0 \Rightarrow$$

$$\psi_1 = A_1 \exp(jK_1 x) + B_1 \exp(-jK_1 x)$$

(incident wave) (reflected wave)

where $K_1 = \sqrt{\dfrac{2mE}{\hbar^2}}$

Region II: $V = V_1$

$$\frac{\partial^2\psi_2}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2 = 0 \Rightarrow$$

$$\psi_2 = A_2 \exp(jK_2 x) + B_2 \exp(-jK_2 x)$$

(transmitted wave) (reflected wave)

where $K_2 = \sqrt{\dfrac{2m(E - V_1)}{\hbar^2}}$

Region III: $V = V_2$

$$\frac{\partial^2\psi_3}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3 = 0 \Rightarrow$$

$$\psi_3 = A_3 \exp(jK_3 x)$$

(transmitted wave)

where $K_3 = \sqrt{\dfrac{2m(E - V_2)}{\hbar^2}}$

There is no reflected wave in region III.

The transmission coefficient is defined as

$$T = \frac{v_3}{v_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*} = \frac{K_3}{K_1} \cdot \frac{A_3 A_3^*}{A_1 A_1^*}$$

From boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow A_1 + B_1 = A_2 + B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 A_1 - K_1 B_1 = K_2 A_2 - K_2 B_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jK_2 a) + B_2 \exp(-jK_2 a) = A_3 \exp(jK_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$K_2 A_2 \exp(jK_2 a) - K_2 B_2 \exp(-jK_2 a) = K_3 A_3 \exp(jK_3 a)$$

But $K_2 a = 2n\pi \Rightarrow$

$$\exp(jK_2 a) = \exp(-jK_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the above equations, we have

$$T = \frac{K_3}{K_1} \cdot \frac{4K_1^2}{(K_1 + K_3)^2} \Rightarrow T = \frac{4K_1 K_3}{(K_1 + K_3)^2}$$

2.37

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2} \psi_1 = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_2 = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1 = B_1 \exp(+K_1 x)$$

$$\psi_2 = A_2 \sin(K_2 x) + B_2 \cos(K_2 x)$$

$$\psi_3 = 0$$

where

$$K_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad K_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b)

Boundary conditions:

$x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow K_1 B_1 = K_2 A_2$$

$x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin K_2 a + B_2 \cos K_2 a = 0$$

or

$$B_2 = -A_2 \tan K_2 a$$

(c)

$$K_1 B_1 = K_2 A_2 \Rightarrow A_2 = \left(\frac{K_1}{K_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{K_1}{K_2}\right) B_2$$

From $B_2 = -A_2 \tan K_2 a$, we can write

$$B_2 = -\left(\frac{K_1}{K_2}\right) B_2 \tan K_2 a$$

which gives

$$1 = -\left(\frac{K_1}{K_2}\right) \tan K_2 a$$

In turn, this equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.38

$$E_n = \frac{-m_o e^4}{(4\pi \epsilon_o)^2 2\hbar^2 n^2} \, (J)$$

$$= \frac{m_o e^3}{(4\pi \epsilon_o)^2 2\hbar^2 n^2} \, (eV)$$

$$= \frac{-(9.11x10^{-31})(1.6x10^{-19})^3}{\left[4\pi(8.85x10^{-12})\right]^2 2(1.054x10^{-34})^2 n^2} \Rightarrow$$

$$E_n = \frac{-13.58}{n^2} \, (eV)$$

Then

$n = 1 \Rightarrow \quad \underline{E_1 = -13.58\ eV}$

$n = 2 \Rightarrow \quad \underline{E_2 = -3.395\ eV}$

$n = 3 \Rightarrow \quad \underline{E_3 = -1.51\ eV}$

$n = 4 \Rightarrow \quad \underline{E_4 = -0.849\ eV}$

2.39

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^* = 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3} \left\{ r^2 \left(\frac{-2}{a_o}\right) \exp\left(\frac{-2r}{a_o}\right) + 2r \exp\left(\frac{-2r}{a_o}\right) \right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow \quad \underline{r = a_o}$$

or $r = a_o$ is the radius that gives the greatest probability.

2.40

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}(E - V(r))\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi \in_o r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) \Rightarrow$$

$$\frac{d\psi_{100}}{dr} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left(\frac{-1}{a_o}\right) \exp\left(\frac{-r}{a_o}\right)$$

Then

$$r^2 \frac{d\psi_{100}}{dr} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

so that

$$\frac{d}{dr}\left(r^2 \frac{d\psi_{100}}{dr}\right)$$

$$= \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \left[2r \exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right) \exp\left(\frac{-r}{a_o}\right) \right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \left[2r \exp\left(\frac{-r}{a_o}\right) - \frac{r^2}{a_o} \exp\left(\frac{-r}{a_o}\right) \right]$$

$$+ \frac{2m_o}{\hbar^2}\left[E + \frac{\hbar^2}{m_o a_o r}\right] \cdot \left(\frac{1}{\sqrt{\pi}}\right) \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right) = 0$$

where

$$E = E_1 = \frac{-m_o e^4}{(4\pi \in_o)^2 \cdot 2\hbar^2} \Rightarrow \quad E_1 = \frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left[\exp\left(\frac{-r}{a_o}\right)\right] \left\{ \frac{-1}{r^2 a_o} \left[2r - \frac{r^2}{a_o} \right] \right.$$

$$\left. + \frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o} + \frac{\hbar^2}{m_o a_o r}\right) \right\} = 0$$

or

$$\frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \left[\exp\left(\frac{-r}{a_o}\right)\right]$$

$$\times \left\{ \frac{-2}{a_o r} + \frac{1}{a_o^2} + \left(\frac{-1}{a_o^2} + \frac{2}{a_o r}\right) \right\} = 0$$

which gives 0 = 0, and shows that ψ_{100} is indeed a solution of the wave equation.

2.41

All elements from Group I column of the periodic table. All have one valence electron in the outer shell.