Chapter 4

Problem Solutions

4.1

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

(a) Silicon

$T(°K)$	$kT\ (eV)$	$n_i(cm^{-3})$
200	0.01727	$7.68x10^4$
400	0.03453	$2.38x10^{12}$
600	0.0518	$9.74x10^{14}$

(b) Germanium (c) GaAs

$T(°K)$	$n_i(cm^{-3})$	$n_i(cm^{-3})$
200	$2.16x10^{10}$	1.38
400	$8.60x10^{14}$	$3.28x10^9$
600	$3.82x10^{16}$	$5.72x10^{12}$

4.2

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

$$\left(10^{12}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \exp\left(\frac{-1.12}{kT}\right)$$

Then

$$\exp\left(\frac{1.12}{kT}\right) = \left(2.912x10^{14}\right)\left(\frac{T}{300}\right)^3$$

By trial and error

$$\underline{T = 381K}$$

4.3

Computer Plot

4.4

$$n_i^2 = N_{CO}N_{VO} \cdot (T)^3 \cdot \exp\left(\frac{-E_g}{kT}\right)$$

So

$$\frac{n_i^2(T_2)}{n_i^2(T_1)} = \left(\frac{T_2}{T_1}\right)^3 \exp\left[-E_g\left(\frac{1}{kT_2} - \frac{1}{kT_1}\right)\right]$$

At $T_2 = 300K \Rightarrow kT = 0.0259\ eV$

At $T_1 = 200K \Rightarrow kT = 0.01727\ eV$

Then

$$\left(\frac{5.83x10^7}{1.82x10^2}\right)^2 = \left(\frac{300}{200}\right)^3 \exp\left[-E_g\left(\frac{1}{0.0259} - \frac{1}{0.01727}\right)\right]$$

or

$$1.026x10^{11} = 3.375\exp\left[(19.29)E_g\right]$$

which yields

$$\underline{E_g = 1.25\ eV}$$

For $T = 300K$,

$$\left(5.83x10^7\right)^2 = \left(N_{CO}N_{VO}\right)(300)^3 \exp\left(\frac{-1.25}{0.0259}\right)$$

or

$$\underline{N_{CO}N_{VO} = 1.15x10^{29}}$$

4.5

(a) $g_C f_F \propto \sqrt{E - E_C}\exp\left[\frac{-(E - E_F)}{kT}\right]$

$$\propto \sqrt{E - E_C}\exp\left[\frac{-(E - E_C)}{kT}\right]\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

Let $E - E_C \equiv x$

Then

$$g_C f_F \propto \sqrt{x}\exp\left(\frac{-x}{kT}\right)$$

Now, to find the maximum value

$$\frac{d(g_C f_F)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right) - \frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right) = 0$$

This yields

$$\frac{1}{2x^{1/2}} = \frac{x^{1/2}}{kT} \Rightarrow x = \frac{kT}{2}$$

Then the maximum value occurs at

$$\underline{E = E_C + \frac{kT}{2}}$$

(b)

$$g_V(1 - f_F) \propto \sqrt{E_V - E}\exp\left[\frac{-(E_F - E)}{kT}\right]$$

$$\propto \sqrt{E_V - E}\exp\left[\frac{-(E_F - E_V)}{kT}\right]\exp\left[\frac{-(E_V - E)}{kT}\right]$$

Let $E_V - E \equiv x$

Then

$$g_V(1-f_F) \propto \sqrt{x}\exp\left(\frac{-x}{kT}\right)$$

To find the maximum value

$$\frac{d\left[g_V(1-f_F)\right]}{dx} \propto \frac{d}{dx}\left[\sqrt{x}\exp\left(\frac{-x}{kT}\right)\right] = 0$$

Same as part (a). Maximum occurs at

$$x = \frac{kT}{2} = E_V - E$$

or

$$\underline{E = E_V - \frac{kT}{2}}$$

4.6

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{E_1 - E_C}\exp\left[\frac{-(E_1 - E_C)}{kT}\right]}{\sqrt{E_2 - E_C}\exp\left[\frac{-(E_2 - E_C)}{kT}\right]}$$

where

$$E_1 = E_C + 4kT \quad \text{and} \quad E_2 = E_C + \frac{kT}{2}$$

Then

$$\frac{n(E_1)}{n(E_2)} = \frac{\sqrt{4kT}}{\sqrt{\frac{kT}{2}}}\exp\left[\frac{-(E_1 - E_2)}{kT}\right]$$

$$= 2\sqrt{2}\exp\left[-\left(4 - \frac{1}{2}\right)\right] = 2\sqrt{2}\exp(-3.5)$$

or

$$\underline{\frac{n(E_1)}{n(E_2)} = 0.0854}$$

4.7
Computer Plot

4.8

$$\frac{n_i^2(A)}{n_i^2(B)} = \frac{\exp\left(\frac{-E_{gA}}{kT}\right)}{\exp\left(\frac{-E_{gB}}{kT}\right)} = \exp\left[\frac{-(E_{gA} - E_{gB})}{kT}\right]$$

or

$$\frac{n_i(A)}{n_i(B)} = \exp\left[\frac{-(E_{gA} - E_{gB})}{2kT}\right]$$

$$= \exp\left[\frac{-(1 - 1.2)}{2(0.0259)}\right] = \exp\left[\frac{+0.20}{2(0.0259)}\right]$$

or

$$\underline{\frac{n_i(A)}{n_i(B)} = 47.5}$$

4.9
Computer Plot

4.10

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

Silicon: $m_p^* = 0.56m_O$, $m_n^* = 1.08m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0128\ eV}$$

Germanium: $m_p^* = 0.37m_O$, $m_n^* = 0.55m_O$

$$\underline{E_{Fi} - E_{midgap} = -0.0077\ eV}$$

Gallium Arsenide: $m_p^* = 0.48m_O$, $m_n^* = 0.067m_O$

$$\underline{E_{Fi} - E_{midgap} = +0.038\ eV}$$

4.11

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln\left(\frac{1.4}{0.62}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = +0.0158\ eV}$$

(b)

$$E_{Fi} - E_{midgap} = \frac{3}{4}(0.0259)\ln\left(\frac{0.25}{1.10}\right) \Rightarrow$$

$$\underline{E_{Fi} - E_{midgap} = -0.0288\ eV}$$

4.12

$$E_{Fi} - E_{midgap} = \frac{1}{2}(kT)\ln\left(\frac{N_V}{N_C}\right)$$

$$= \frac{1}{2}(kT)\ln\left(\frac{1.04x10^{19}}{2.8x10^{19}}\right) = -0.495(kT)$$

$T(°K)$	$kT\ (eV)$	$E_{Fi} - E_{midgap}\ (eV)$
200	0.01727	−0.0085
400	0.03453	−0.017
600	0.0518	−0.0256

4.13
Computer Plot

4.14
Let $g_C(E) = K =$ constant
Then,

$$n_o = \int_{E_C}^{\infty} g_C(E) f_F(E)\, dE$$

$$= K \int_{E_C}^{\infty} \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$\approx K \int_{E_C}^{\infty} \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_F}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$E - E_F = (E_C - E_F) - (E_C - E)$$

so that

$$\exp\left[\frac{-(E-E_F)}{kT}\right] = \exp\left[\frac{-(E_C-E_F)}{kT}\right] \cdot \exp(-\eta)$$

The integral can then be written as

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right] \int_0^{\infty} \exp(-\eta)\, d\eta$$

which becomes

$$n_o = K \cdot kT \cdot \exp\left[\frac{-(E_C-E_F)}{kT}\right]$$

4.15
Let $g_C(E) = C_1(E - E_C)$ for $E \ge E_C$

$$n_o = \int_{E_C}^{\infty} g_C(E) f_F(E)\, dE$$

$$= C_1 \int_{E_C}^{\infty} \frac{(E-E_C)}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

or

$$n_o \approx C_1 \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_F)}{kT}\right] dE$$

Let

$$\eta = \frac{E-E_C}{kT} \text{ so that } dE = kT \cdot d\eta$$

We can write

$$(E - E_F) = (E - E_C) + (E_C - E_F)$$

Then

$$n_o = C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_{E_C}^{\infty} (E-E_C) \exp\left[\frac{-(E-E_C)}{kT}\right] dE$$

or

$$= C_1 \exp\left[\frac{-(E_C-E_F)}{kT}\right] \times \int_0^{\infty} (kT)\eta[\exp(-\eta)](kT)\, d\eta$$

We find that

$$\int_0^{\infty} \eta \exp(-\eta)\, d\eta = \frac{e^{-\eta}}{1}(-\eta - 1)\Big|_0^{\infty} = +1$$

So

$$n_o = C_1 (kT)^2 \exp\left[\frac{-(E_C-E_F)}{kT}\right]$$

4.16
We have $\frac{r_1}{a_o} = \in_r \left(\frac{m_o}{m^*}\right)$

For Germanium, $\in_r = 16$, $m^* = 0.55 m_o$
Then

$$r_1 = (16)\left(\frac{1}{0.55}\right) a_o = 29(0.53)$$

so

$$r_1 = 15.4\ \mathring{A}$$

The ionization energy can be written as

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) \quad eV$$

$$= \frac{0.55}{(16)^2}(13.6) \Rightarrow \underline{E = 0.029\ eV}$$

4.17

We have $\frac{r_1}{a_o} = \in_r \left(\frac{m_o}{m^*}\right)$

For GaAs, $\in_r = 13.1$, $m^* = 0.067 m_o$

Then

$$r_1 = (13.1)\left(\frac{1}{0.067}\right)(0.53)$$

or

$$\underline{r_1 = 104\ \overset{\circ}{A}}$$

The ionization energy is

$$E = \left(\frac{m^*}{m_o}\right)\left(\frac{\in_o}{\in_s}\right)^2 (13.6) = \frac{0.067}{(13.1)^2}(13.6)$$

or

$$\underline{E = 0.0053\ eV}$$

4.18

(a) $p_o = \frac{n_i^2}{n_o} = \frac{(1.5x10^{10})^2}{5x10^4} \Rightarrow$

$\underline{p_o = 4.5x10^{15}\ cm^{-3}}$, $p_o > n_o \Rightarrow$ p-type

(b)

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{4.5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3266\ eV}$$

4.19

$$p_o = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$= 1.04x10^{19} \exp\left(\frac{-0.22}{0.0259}\right)$$

so

$$\underline{p_o = 2.13x10^{15}\ cm^{-3}}$$

Assuming

$E_C - E_F = 1.12 - 0.22 = 0.90\ eV$

Then

$$n_o = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{18} \exp\left(\frac{-0.90}{0.0259}\right)$$

or

$$\underline{n_o = 2.27x10^4\ cm^{-3}}$$

4.20

(a) $T = 400K \Rightarrow kT = 0.03453\ eV$

$$N_C = 4.7x10^{17}\left(\frac{400}{300}\right)^{3/2} = 7.24x10^{17}\ cm^{-3}$$

Then

$$n_o = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 7.24x10^{17} \exp\left(\frac{-0.25}{0.03453}\right)$$

or

$$\underline{n_o = 5.19x10^{14}\ cm^{-3}}$$

Also

$$N_V = 7x10^{18}\left(\frac{400}{300}\right)^{3/2} = 1.08x10^{19}\ cm^{-3}$$

and

$E_F - E_V = 1.42 - 0.25 = 1.17\ eV$

Then

$$p_o = 1.08x10^{19} \exp\left(\frac{-1.17}{0.03453}\right)$$

or

$$\underline{p_o = 2.08x10^4\ cm^{-3}}$$

(b)

$$E_C - E_F = kT \ln\left(\frac{N_C}{n_o}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5.19x10^{14}}\right)$$

or $\underline{E_C - E_F = 0.176\ eV}$

Then

$E_F - E_V = 1.42 - 0.176 = 1.244\ eV$

and

$$p_o = (7x10^{18})\exp\left(\frac{-1.244}{0.0259}\right)$$

or $\underline{p_o = 9.67x10^{-3}\ cm^{-3}}$

4.21

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

or

$$E_F - E_V = kT \ln\left(\frac{N_V}{p_O}\right)$$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{10^{15}}\right) = 0.24\ eV$$

Then

$$E_C - E_F = 1.12 - 0.24 = 0.88\ eV$$

So

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$= 2.8x10^{19} \exp\left(\frac{-0.88}{0.0259}\right)$$

or

$$\underline{n_O = 4.9x10^4\ cm^{-3}}$$

4.22

(a) $p_O = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$

$$1.5x10^{10} \exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.11x10^{16}\ cm^{-3}}$$

(b)

From Problem 4.1, $n_i(400K) = 2.38x10^{12}\ cm^{-3}$

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453\ eV$$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.11x10^{16}}{2.38x10^{12}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.292\ eV}$$

(c)

From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 2.03x10^4\ cm^{-3}}$$

From (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.38x10^{12})^2}{1.11x10^{16}}$$

or

$$\underline{n_O = 5.10x10^8\ cm^{-3}}$$

4.23

(a) $p_O = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$

$$= (1.8x10^6)\exp\left(\frac{0.35}{0.0259}\right)$$

or

$$\underline{p_O = 1.33x10^{12}\ cm^{-3}}$$

(b) From Problem 4.1,

$n_i(400K) = 3.28x10^9\ cm^{-3}$, $kT = 0.03453\ eV$

Then

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

$$= (0.03453)\ln\left(\frac{1.33x10^{12}}{3.28x10^9}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.207\ eV}$$

(c) From (a)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.8x10^6)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 2.44\ cm^{-3}}$$

From (b)

$$n_O = \frac{(3.28x10^9)^2}{1.33x10^{12}}$$

or

$$\underline{n_O = 8.09x10^6\ cm^{-3}}$$

4.24

For silicon, $T = 300K$, $E_F = E_V$

$$\eta' = \frac{E_V - E_F}{kT} = 0 \Rightarrow F_{1/2}(\eta') = 0.60$$

We can write

$$p_O = \frac{2}{\sqrt{\pi}} N_V F_{1/2}(\eta') = \frac{2}{\sqrt{\pi}}\left(1.04x10^{19}\right)(0.60)$$

or

$$p_O = 7.04x10^{18}\ cm^{-3}$$

4.25

Silicon, $T = 300K$, $n_O = 5x10^{19}\ cm^{-3}$

We have

$$n_O = \frac{2}{\sqrt{\pi}} N_C F_{1/2}(\eta_F)$$

or

$$5x10^{19} = \frac{2}{\sqrt{\pi}}\left(2.8x10^{19}\right)F_{1/2}(\eta_F)$$

which gives

$$F_{1/2}(\eta_F) = 1.58$$

Then

$$\eta_F = 1.3 = \frac{E_F - E_C}{kT}$$

or $E_F - E_C = (1.3)(0.0259) \Rightarrow$

$$E_C - E_F = -0.034\ eV$$

4.26

For the electron concentration

$$n(E) = g_C(E) f_F(E)$$

The Boltzmann approximation applies so

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}\ \exp\left[\frac{-(E - E_F)}{kT}\right]$$

or

$$n(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E - E_C}{kT}}\ \exp\left[\frac{-(E - E_C)}{kT}\right]$$

Define

$$x = \frac{E - E_C}{kT}$$

Then

$$n(E) \to n(x) = K\sqrt{x}\ \exp(-x)$$

To find maximum $n(E) \to n(x)$, set

$$\frac{dn(x)}{dx} = 0 = K\left[\frac{1}{2}x^{-1/2}\exp(-x) + x^{1/2}(-1)\exp(-x)\right]$$

or

$$0 = Kx^{-1/2}\exp(-x)\left[\frac{1}{2} - x\right]$$

which yields

$$x = \frac{1}{2} = \frac{E - E_C}{kT} \Rightarrow \quad E = E_C + \frac{1}{2}kT$$

For the hole concentration

$$p(E) = g_V(E)\left[1 - f_F(E)\right]$$

From the text, using the Maxwell-Boltzmann approximation, we can write

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_V - E}\ \exp\left[\frac{-(E_F - E)}{kT}\right]$$

or

$$p(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

$$\times\sqrt{kT}\sqrt{\frac{E_V - E}{kT}}\ \exp\left[\frac{-(E_V - E)}{kT}\right]$$

Define $x' = \dfrac{E_V - E}{kT}$

Then

$$p(x') = K'\sqrt{x'}\ \exp(-x')$$

To find the maximum of $p(E) \to p(x')$, set $\dfrac{dp(x')}{dx'} = 0$. Using the results from above, we find the maximum at

$$E = E_V - \frac{1}{2}kT$$

4.27

(a) Silicon: We have

$$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

We can write

$$E_C - E_F = (E_C - E_d) + (E_d - E_F)$$

For

$$E_C - E_d = 0.045\ eV,\ E_d - E_F = 3kT$$

$$n_O = \left(2.8x10^{19}\right)\exp\left[\frac{-0.045}{0.0259} - 3\right]$$

$$= \left(2.8x10^{19}\right)\exp(-4.737)$$

or

$$n_O = 2.45x10^{17}\ cm^{-3}$$

We also have

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$$

Again, we can write

$$E_F - E_V = (E_F - E_a) + (E_a - E_V)$$

For

$$E_F - E_a = 3kT\,,\ E_a - E_V = 0.045\ eV$$

Then

$$p_O = (1.04x10^{19})\exp\left[-3 - \frac{0.045}{0.0259}\right]$$

$$= (1.04x10^{19})\exp(-4.737)$$

or

$$\underline{p_O = 9.12x10^{16}\ cm^{-3}}$$

(b)
GaAs: Assume $E_C - E_d = 0.0058\ eV$

Then

$$n_O = (4.7x10^{17})\exp\left[\frac{-0.0058}{0.0259} - 3\right]$$

$$= (4.7x10^{17})\exp(-3.224)$$

or

$$\underline{n_O = 1.87x10^{16}\ cm^{-3}}$$

Assume $E_a - E_V = 0.0345\ eV$

Then

$$p_O = (7x10^{18})\exp\left[\frac{-0.0345}{0.0259} - 3\right]$$

$$= (7x10^{18})\exp(-4.332)$$

or

$$\underline{p_O = 9.20x10^{16}\ cm^{-3}}$$

4.28
Computer Plot

4.29
(a) Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Then

$$p_O = \frac{10^{13}}{2} + \sqrt{\left(\frac{10^{13}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{p_O = 2.95x10^{13}\ cm^{-3}}$$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{(2.4x10^{13})^2}{2.95x10^{13}} \Rightarrow$$

$$\underline{n_O = 1.95x10^{13}\ cm^{-3}}$$

(b)

$$n_O = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

Then

$$n_O = \frac{5x10^{15}}{2} + \sqrt{\left(\frac{5x10^{15}}{2}\right)^2 + (2.4x10^{13})^2}$$

or

$$\underline{n_O \approx 5x10^{15}\ cm^{-3}}$$

and

$$p_O = \frac{n_i^2}{n_O} = \frac{(2.4x10^{13})^2}{5x10^{15}} \Rightarrow$$

$$\underline{p_O = 1.15x10^{11}\ cm^{-3}}$$

4.30
For the donor level

$$\frac{n_d}{N_d} = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$= \frac{1}{1 + \frac{1}{2}\exp\left(\frac{0.20}{0.0259}\right)}$$

or

$$\underline{\frac{n_d}{N_d} = 8.85x10^{-4}}$$

And

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Now

$$E - E_F = (E - E_C) + (E_C - E_F)$$

or

$$E - E_F = kT + 0.245$$

Then

$$f_F(E) = \frac{1}{1 + \exp\left(1 + \frac{0.245}{0.0259}\right)} \Rightarrow$$

$$\underline{f_F(E) = 2.87x10^{-5}}$$

4.31

(a) $n_o = N_d = 2x10^{15}\ cm^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5x10^{10}\right)^2}{2x10^{15}} \Rightarrow$$

$p_o = 1.125x10^5\ cm^{-3}$

(b)

$p_o = N_a = 10^{16}\ cm^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} \Rightarrow$$

$n_o = 2.25x10^4\ cm^{-3}$

(c)

$n_o = p_o = n_i = 1.5x10^{10}\ cm^{-3}$

(d)

$T = 400K \Rightarrow kT = 0.03453\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.12}{0.03453}\right)$$

or

$n_i = 2.38x10^{12}\ cm^{-3}$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(2.38x10^{12}\right)^2}$$

or

$p_o = 1.0x10^{14}\ cm^{-3}$

Also

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(2.38x10^{12}\right)^2}{10^{14}} \Rightarrow$$

$n_o = 5.66x10^{10}\ cm^{-3}$

(e)

$T = 500K \Rightarrow kT = 0.04317\ eV$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.12}{0.04317}\right)$$

or

$n_i = 8.54x10^{13}\ cm^{-3}$

Now

$$n_o = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 5x10^{13} + \sqrt{\left(5x10^{13}\right)^2 + \left(8.54x10^{13}\right)^2}$$

or

$n_o = 1.49x10^{14}\ cm^{-3}$

Also

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(8.54x10^{13}\right)^2}{1.49x10^{14}} \Rightarrow$$

$p_o = 4.89x10^{13}\ cm^{-3}$

4.32

(a) $n_o = N_d = 2x10^{15}\ cm^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.8x10^6\right)^2}{2x10^{15}} \Rightarrow$$

$p_o = 1.62x10^{-3}\ cm^{-3}$

(b)

$p_o = N_a = 10^{16}\ cm^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.8x10^6\right)^2}{10^{16}} \Rightarrow$$

$n_o = 3.24x10^{-4}\ cm^{-3}$

(c)

$n_o = p_o = n_i = 1.8x10^6\ cm^{-3}$

(d)

$kT = 0.03453\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{400}{300}\right)^3 \exp\left(\frac{-1.42}{0.03453}\right)$$

or

$n_i = 3.28x10^9\ cm^{-3}$

Now

$p_o = N_a = 10^{14}\ cm^{-3}$

and

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(3.28x10^9\right)^2}{10^{14}} \Rightarrow$$

$n_o = 1.08x10^5\ cm^{-3}$

(e)

$kT = 0.04317\ eV$

$$n_i^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{500}{300}\right)^3 \exp\left(\frac{-1.42}{0.04317}\right)$$

or

$n_i = 2.81x10^{11}\ cm^{-3}$

Now

$$n_O = N_d = 10^{14}\ cm^{-3}$$

Also

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(2.81x10^{11}\right)^2}{10^{14}} \Rightarrow$$

$$p_O = 7.90x10^8\ cm^{-3}$$

4.33

(a) $N_a > N_d \Rightarrow$ p-type

(b) Si:

$$p_O = N_a - N_d = 2.5x10^{13} - 1x10^{13}$$

or

$$p_O = 1.5x10^{13}\ cm^{-3}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{1.5x10^{13}} \Rightarrow$$

$$n_O = 1.5x10^7\ cm^{-3}$$

Ge:

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \left(\frac{1.5x10^{13}}{2}\right) + \sqrt{\left(\frac{1.5x10^{13}}{2}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$p_O = 3.26x10^{13}\ cm^{-3}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(2.4x10^{13}\right)^2}{3.26x10^{13}} \Rightarrow$$

$$n_O = 1.77x10^{13}\ cm^{-3}$$

GaAs:

$$p_O = 1.5x10^{13}\ cm^{-3}$$

And

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.8x10^6\right)^2}{1.5x10^{13}} \Rightarrow$$

$$n_O = 0.216\ cm^{-3}$$

4.34

For $T = 450K$

$$n_i^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{450}{300}\right)^3 \times \exp\left[\frac{-1.12}{(0.0259)(450/300)}\right]$$

or

$$n_i = 1.72x10^{13}\ cm^{-3}$$

(a)

$N_a > N_d \Rightarrow$ p-type

(b)

$$p_O = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

$$= \frac{1.5x10^{15} - 8x10^{14}}{2} + \sqrt{\left(\frac{1.5x10^{15} - 8x10^{14}}{2}\right)^2 + \left(1.72x10^{13}\right)^2}$$

or

$$p_O \approx N_a - N_d = 7x10^{14}\ cm^{-3}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.72x10^{13}\right)^2}{7x10^{14}} \Rightarrow$$

$$n_O = 4.23x10^{11}\ cm^{-3}$$

(c)

Total ionized impurity concentration

$$N_I = N_a + N_d = 1.5x10^{15} + 8x10^{14}$$

or

$$N_I = 2.3x10^{15}\ cm^{-3}$$

4.35

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{2x10^5} \Rightarrow$$

$$n_O = 1.125x10^{15}\ cm^{-3}$$

$n_O > p_O \Rightarrow$ n-type

4.36

$$kT = (0.0259)\left(\frac{200}{300}\right) = 0.01727\ eV$$

$$n_i^2 = (4.7x10^{17})(7x10^{18})\left(\frac{200}{300}\right)^3 \times \exp\left[\frac{-1.42}{0.01727}\right]$$

or

$$\underline{n_i = 1.38\ cm^{-3}}$$

Now

$$n_O p_O = n_i^2 \Rightarrow 5p_O^2 = n_i^2$$

or\

$$p_O = \frac{n_i}{\sqrt{5}} \Rightarrow \underline{p_O = 0.617\ cm^{-3}}$$

And

$$n_O = 5p_O \Rightarrow \underline{n_O = 3.09\ cm^{-3}}$$

4.37
Computer Plot

4.38
Computer Plot

4.39
Computer Plot

4.40
n-type, so majority carrier = electrons

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{(10^{13})^2 + (2x10^{13})^2}$$

or

$$\underline{n_O = 3.24x10^{13}\ cm^{-3}}$$

Then

$$p_O = \frac{n_i^2}{n_O} = \frac{(2x10^{13})^2}{3.24x10^{13}} \Rightarrow$$

$$\underline{p_O = 1.23x10^{13}\ cm^{-3}}$$

4.41

(a) $N_d > N_a \Rightarrow$ n-type

$$n_O = N_d - N_a = 2x10^{16} - 1x10^{16}$$

or

$$\underline{n_O = 1x10^{16}\ cm^{-3}}$$

Then

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{16}} \Rightarrow$$

$$\underline{p_O = 2.25x10^4\ cm^{-3}}$$

(b)

$N_a > N_d \Rightarrow$ p-type

$$p_O = N_a - N_d = 3x10^{16} - 2x10^{15}$$

or

$$\underline{p_O = 2.8x10^{16}\ cm^{-3}}$$

Then

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{2.8x10^{16}} \Rightarrow$$

$$\underline{n_O = 8.04x10^3\ cm^{-3}}$$

4.42

(a) $n_O < n_i \Rightarrow$ p-type

(b) $\underline{n_O = 4.5x10^4\ cm^{-3}} \Rightarrow$ electrons: minority carrier

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{4.5x10^4} \Rightarrow$$

$\underline{p_O = 5x10^{15}\ cm^{-3}} \Rightarrow$ holes: majority carrier

(c)

$$p_O = N_a - N_d$$

so

$$5x10^{15} = N_a - 5x10^{15} \Rightarrow \underline{N_a = 10^{16}\ cm^{-3}}$$

Acceptor impurity concentration,

$\underline{N_d = 5x10^{15}\ cm^{-3}}$ Donor impurity concentration

4.43

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

For Germanium:

$T(°K)$	$kT(eV)$	$n_i(cm^{-3})$
200	0.01727	$2.16x10^{10}$
400	0.03454	$8.6x10^{14}$
600	0.0518	$3.82x10^{16}$

$$p_O = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \text{ and } N_a = 10^{15} \ cm^{-3}$$

$T(°K)$	$p_O(cm^{-3})$	$E_{Fi} - E_F \ (eV)$
200	$1.0x10^{15}$	0.1855
400	$1.49x10^{15}$	0.01898
600	$3.87x10^{16}$	0.000674

4.44

$$E_F - E_{Fi} = kT \ln\left(\frac{n_O}{n_i}\right)$$

For Germanium,

$$T = 300K \Rightarrow n_i = 2.4x10^{13} \ cm^{-3}$$

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$N_d(cm^{-3})$	$n_O(cm^{-3})$	$E_F - E_{Fi} \ (eV)$
10^{14}	$1.05x10^{14}$	0.0382
10^{16}	10^{16}	0.156
10^{18}	10^{18}	0.2755

4.45

$$n_O = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

Now

$$n_i = 0.05 n_O$$

so

$$n_O = 1.5x10^{15} + \sqrt{\left(1.5x10^{15}\right)^2 + \left[(0.05)n_O\right]^2}$$

which yields

$$n_O = 3.0075x10^{15} \ cm^{-3}$$

Then

$$n_i = 1.504x10^{14} \ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

so

$$\left(1.504x10^{14}\right)^2 = \left(4.7x10^{17}\right)\left(7x10^{18}\right)\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.42}{(0.0259)(T/300)}\right]$$

By trial and error

$$T \approx 762K$$

4.46

Computer Plot

4.47

Computer Plot

4.48

(a) $E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$

$$= \frac{3}{4}(0.0259)\ln(10) \Rightarrow$$

$$E_{Fi} - E_{midgap} = +0.0447 \ eV$$

(b)

Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45 \ eV$$

(i) p-type, so add acceptor impurities

(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947 \ eV$

$$p_O = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) = 10^5 \exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_O = N_a = 1.97x10^{13} \ cm^{-3}$$

4.49

$$n_O = N_d - N_a = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

so

$$N_d = 5x10^{15} + 2.8x10^{19} \exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5x10^{15} + 6.95x10^{15}$$

so

$$N_d = 1.2x10^{16} \ cm^{-3}$$

4.50

(a) $p_O = N_a = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right]$

or

$$\exp\left[\frac{+(E_F - E_V)}{kT}\right] = \frac{N_V}{N_a} = \frac{1.04x10^{19}}{7x10^{15}}$$
$$= 1.49x10^3$$

Then

$$E_F - E_V = (0.0259)\ln(1.49x10^3)$$

or

$$\underline{E_F - E_V = 0.189\ eV}$$

(b)

If $E_F - E_V = 0.1892 - 0.0259 = 0.1633\ eV$

Then

$$N_a = 1.04x10^{19}\exp\left(\frac{-0.1633}{0.0259}\right)$$
$$= 1.90x10^{16}\ cm^{-3}$$

so that

$$\Delta N_a = 1.90x10^{16} - 7x10^{15} \Rightarrow$$
$$\underline{\Delta N_a = 1.2x10^{16}\ cm^{-3}}$$

Acceptor impurities to be added

4.51

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = 0.2877\ eV$$

(c)

For (a), $\underline{n_O = N_d = 10^{15}\ cm^{-3}}$

For (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$
$$\underline{n_O = 2.25x10^5\ cm^{-3}}$$

4.52

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{p_O}{n_i}\right) = 0.45\ eV$$

Then

$$p_O = (1.8x10^6)\exp\left(\frac{0.45}{0.0259}\right) \Rightarrow$$
$$\underline{p_O = 6.32x10^{13}\ cm^{-3}}$$

Now

$p_O < N_a$, Donors must be added

$$p_O = N_a - N_d \Rightarrow N_d = N_a - p_O$$

so

$$N_d = 10^{15} - 6.32x10^{13} \Rightarrow$$
$$\underline{N_d = 9.368x10^{14}\ cm^{-3}}$$

4.53

(a) $E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$
$$\underline{E_F - E_{Fi} = 0.3056\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) \Rightarrow$$
$$\underline{E_{Fi} - E_F = 0.3473\ eV}$$

(c)

$$\underline{E_F = E_{Fi}}$$

(d)

$kT = 0.03453\ eV$, $n_i = 2.38x10^{12}\ cm^{-3}$

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right)$$
$$= (0.03453)\ln\left(\frac{10^{14}}{2.38x10^{12}}\right) \Rightarrow$$
$$\underline{E_{Fi} - E_F = 0.1291\ eV}$$

(e)

$kT = 0.04317\ eV$, $n_i = 8.54x10^{13}\ cm^{-3}$

$$E_F - E_{Fi} = kT\ln\left(\frac{n_O}{n_i}\right)$$
$$= (0.04317)\ln\left(\frac{1.49x10^{14}}{8.54x10^{13}}\right) \Rightarrow$$
$$\underline{E_F - E_{Fi} = 0.0024\ eV}$$

4.54

(a) $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{2x10^{15}}{1.8x10^{6}}\right) \Rightarrow$$

$\underline{E_F - E_{Fi} = 0.5395\ eV}$

(b)

$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.8x10^{6}}\right) \Rightarrow$$

$\underline{E_{Fi} - E_F = 0.5811\ eV}$

(c)

$\underline{E_F = E_{Fi}\ E_{Fi} -}$

(d)

$kT = 0.03453\ eV\ ,\ n_i = 3.28x10^{9}\ cm^{-3}$

$E_{Fi} - E_F = (0.03453)\ln\left(\frac{10^{14}}{3.28x10^{9}}\right) \Rightarrow$

$\underline{E_{Fi} - E_F = 0.3565\ eV}$

(e)

$kT = 0.04317\ eV\ ,\ n_i = 2.81x10^{11}\ cm^{-3}$

$E_F - E_{Fi} = kT \ln\left(\frac{n_O}{n_i}\right)$

$$= (0.04317)\ln\left(\frac{10^{14}}{2.81x10^{11}}\right) \Rightarrow$$

$\underline{E_F - E_{Fi} = 0.2536\ eV}$

4.55

p-type

$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$\underline{E_{Fi} - E_F = 0.3294\ eV}$

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