Chapter 6

Problem Solutions

6.1
n-type semiconductor, low-injection so that

$$R' = \frac{\delta p}{\tau_{pO}} = \frac{5x10^{13}}{10^{-6}}$$

or

$$\underline{R' = 5x10^{19}\ cm^{-3}s^{-1}}$$

6.2

(a) $R_{nO} = \dfrac{n_O}{\tau_{nO}}$

and

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(10^{10}\right)^2}{10^{16}} = 10^4\ cm^{-3}$$

Then

$$R_{nO} = \frac{10^4}{2x10^{-7}} \Rightarrow \underline{R_{nO} = 5x10^{10}\ cm^{-3}s^{-1}}$$

(b)

$$R_n = \frac{\delta n}{\tau_{nO}} = \frac{10^{12}}{2x10^{-7}} \text{ or } R_n = 5x10^{18}\ cm^{-3}s^{-1}$$

so

$$\Delta R_n = R_n - R_{nO} = 5x10^{18} - 5x10^{10} \Rightarrow$$

$$\underline{\Delta R_n \approx 5x10^{18}\ cm^{-3}s^{-1}}$$

6.3
(a) Recombination rates are equal

$$\frac{n_O}{\tau_{nO}} = \frac{p_O}{\tau_{pO}}$$

$$n_O = N_d = 10^{16}\ cm^{-3}$$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

So

$$\frac{10^{16}}{\tau_{nO}} = \frac{2.25x10^4}{20x10^{-6}}$$

or

$$\underline{\tau_{nO} = 8.89x10^{+6}\ s}$$

(b) Generation Rate = Recombination Rate
So

$$G = \frac{2.25x10^4}{20x10^{-6}} \Rightarrow \underline{G = 1.125x10^9\ cm^{-3}s^{-1}}$$

(c)

$$\underline{R = G = 1.125x10^9\ cm^{-3}s^{-1}}$$

6.4

(a) $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{\left(6.625x10^{-34}\right)\left(3x10^8\right)}{6300x10^{-10}}$

or

$E = 3.15x10^{-19}\ J$ This is the energy of 1 photon.
Now

$1\,W = 1\ J/s \Rightarrow\ 3.17x10^{18}$ photons/s

Volume = $(1)(0.1) = 0.1\ cm^{+3}$

Then

$$g = \frac{3.17x10^{18}}{0.1} \Rightarrow$$

$$\underline{g = 3.17x10^{19}\ e-h\ pairs/cm^3-s}$$

(b)

$$\delta n = \delta p = g\tau = \left(3.17x10^{19}\right)\left(10x10^{-6}\right)$$

or

$$\underline{\delta n = \delta p = 3.17x10^{14}\ cm^{-3}}$$

6.5
We have

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

and

$$J_p = e\mu_p p\mathrm{E} - eD_p \nabla p$$

The hole particle current density is

$$F_p^+ = \frac{J_p}{(+e)} = \mu_p p\mathrm{E} - D_p \nabla p$$

Now

$$\nabla \bullet F_p^+ = \mu_p \nabla \bullet (p\mathrm{E}) - D_p \nabla \bullet \nabla p$$

We can write

$$\nabla \bullet (p\mathrm{E}) = \mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}$$

and

$$\nabla \bullet \nabla p = \nabla^2 p$$

so

$$\nabla \bullet F_p^+ = \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) - D_p\nabla^2 p$$

Then

$$\frac{\partial p}{\partial t} = -\mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + D_p\nabla^2 p + g_p - \frac{p}{\tau_p}$$

We can then write

$$D_p\nabla^2 p - \mu_p(\mathrm{E} \bullet \nabla p + p\nabla \bullet \mathrm{E}) + g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}$$

6.6
From Equation [6.18]

$$\frac{\partial p}{\partial t} = -\nabla \bullet F_p^+ + g_p - \frac{p}{\tau_p}$$

For steady-state, $\frac{\partial p}{\partial t} = 0$

Then

$$0 = -\nabla \bullet F_p^+ + g_p - R_p$$

and for a one-dimensional case,

$$\frac{dF_p^+}{dx} = g_p - R_p = 10^{20} - 2x10^{19} \Rightarrow$$

$$\frac{dF_p^+}{dx} = 8x10^{19} \; cm^{-3}s^{-1}$$

6.7
From Equation [6.18],

$$0 = -\frac{dF_p^+}{dx} + 0 - 2x10^{19}$$

or

$$\frac{dF_p^+}{dx} = -2x10^{19} \; cm^{-3}s^{-1}$$

6.8
We have the continuity equations

(1) $$D_p\nabla^2(\delta p) - \mu_p[\mathrm{E} \bullet \nabla(\delta p) + p\nabla \bullet \mathrm{E}] + g_p - \frac{p}{\tau_p} = \frac{\partial(\delta p)}{\partial t}$$

and

(2) $$D_n\nabla^2(\delta n) + \mu_n[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}] + g_n - \frac{n}{\tau_n} = \frac{\partial(\delta n)}{\partial t}$$

By charge neutrality

$$\delta n = \delta p \equiv \delta n \Rightarrow \nabla(\delta n) = \nabla(\delta p)$$

and $\nabla^2(\delta n) = \nabla^2(\delta p)$ and $\frac{\partial(\delta n)}{\partial t} = \frac{\partial(\delta p)}{\partial t}$

Also

$$g_n = g_p \equiv g \,, \; \frac{p}{\tau_p} = \frac{n}{\tau_n} \equiv R$$

Then we can write

(1) $$D_p\nabla^2(\delta n) - \mu_p[\mathrm{E} \bullet \nabla(\delta n) + p\nabla \bullet \mathrm{E}] + g - R = \frac{\partial(\delta n)}{\partial t}$$

and

(2) $$D_n\nabla^2(\delta n) + \mu_n[\mathrm{E} \bullet \nabla(\delta n) + n\nabla \bullet \mathrm{E}] + g - R = \frac{\partial(\delta n)}{\partial t}$$

Multiply Equation (1) by $\mu_n n$ and Equation (2) by $\mu_p p$, and then add the two equations.

We find

$$(\mu_n n D_p + \mu_p p D_n)\nabla^2(\delta n) + \mu_n\mu_p(p-n)\mathrm{E} \bullet \nabla(\delta n) + (\mu_n n + \mu_p p)(g-R) = (\mu_n n + \mu_p p)\frac{\partial(\delta n)}{\partial t}$$

Divide by $(\mu_n n + \mu_p p)$, then

$$\left(\frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}\right)\nabla^2(\delta n) + \left[\frac{\mu_n\mu_p(p-n)}{\mu_n n + \mu_p p}\right]\mathrm{E} \bullet \nabla(\delta n) + (g-R) = \frac{\partial(\delta n)}{\partial t}$$

Define

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} = \frac{D_n D_p(n+p)}{D_n n + D_p p}$$

and $$\mu' = \frac{\mu_n\mu_p(p-n)}{\mu_n n + \mu_p p}$$

Then we have

$$D'\nabla^2(\delta n) + \mu'\mathrm{E} \bullet \nabla(\delta n) + (g-R) = \frac{\partial(\delta n)}{\partial t}$$

Q.E.D.

6.9

For Ge: $T = 300K$, $n_i = 2.4x10^{13}\ cm^{-3}$

$$n = \frac{N_d}{2} + \sqrt{\left(\frac{N_d}{2}\right)^2 + n_i^2}$$

$$= 10^{13} + \sqrt{\left(10^{13}\right)^2 + \left(2.4x10^{13}\right)^2}$$

or

$$n = 3.6x10^{13}\ cm^{-3}$$

Also

$$p = \frac{n_i^2}{n} = \frac{\left(2.4x10^{13}\right)^2}{3.6x10^{13}} = 1.6x10^{13}\ cm^{-3}$$

We have

$$\mu_n = 3900\ ,\ \mu_p = 1900$$

$$D_n = 101\ ,\ D_p = 49.2$$

Now

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}$$

$$= \frac{(101)(49.2)\left(3.6x10^{13} + 1.6x10^{13}\right)}{(101)\left(3.6x10^{13}\right) + (49.2)\left(1.6x10^{13}\right)}$$

or

$$D' = 58.4\ cm^2/s$$

Also

$$\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$$

$$= \frac{(3900)(1900)\left(1.6x10^{13} - 3.6x10^{13}\right)}{(3900)\left(3.6x10^{13}\right) + (1900)\left(1.6x10^{13}\right)}$$

or

$$\mu' = -868\ cm^2/V-s$$

Now

$$\frac{n}{\tau_n} = \frac{p}{\tau_p} \Rightarrow \frac{3.6x10^{13}}{\tau_n} = \frac{1.6x10^{13}}{24\ \mu s}$$

which yields

$$\tau_n = 54\ \mu s$$

6.10

$$\sigma = e\mu_n n + e\mu_p p$$

With excess carriers present

$$n = n_O + \delta n \quad \text{and} \quad p = p_O + \delta p$$

For an n-type semiconductor, we can write

$$\delta n = \delta p \equiv \delta p$$

Then

$$\sigma = e\mu_n\left(n_O + \delta p\right) + e\mu_p\left(p_O + \delta p\right)$$

or

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\Delta\sigma = e\left(\mu_n + \mu_p\right)(\delta p)$$

In steady-state, $\delta p = g'\tau$

So that

$$\Delta\sigma = e\left(\mu_n + \mu_p\right)\left(g'\tau_{pO}\right)$$

6.11

n-type, so that minority carriers are holes. Uniform generation throughout the sample means we have

$$g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

Homogeneous solution is of the form

$$(\delta p)_H = A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

and the particular solution is

$$(\delta p)_P = g'\tau_{pO}$$

so that the total solution is

$$(\delta p) = g'\tau_{pO} + A\exp\left(\frac{-t}{\tau_{pO}}\right)$$

At $t = 0$, $\delta p = 0$ so that

$$0 = g'\tau_{pO} + A \Rightarrow A = -g'\tau_{pO}$$

Then

$$\delta p = g'\tau_{pO}\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

The conductivity is

$$\sigma = e\mu_n n_O + e\mu_p p_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

$$\approx e\mu_n n_O + e\left(\mu_n + \mu_p\right)(\delta p)$$

so

$$\sigma = \left(1.6x10^{-19}\right)(1000)\left(5x10^{16}\right) + \left(1.6x10^{-19}\right)(1000 + 420)\left(5x10^{21}\right)\left(10^{-7}\right) \times \left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

Then

$$\sigma = 8 + 0.114\left[1 - \exp\left(\frac{-t}{\tau_{pO}}\right)\right]$$

where $\tau_{pO} = 10^{-7}\ s$

6.12

n-type GaAs:

$$\Delta\sigma = e(\mu_n + \mu_p)(\delta p)$$

In steady-state, $\delta p = g'\tau_{pO}$. Then

$$\Delta\sigma = (1.6x10^{-19})(8500 + 400)(2x10^{21})(2x10^{-7})$$

or

$$\Delta\sigma = 0.57\ (\Omega - cm)^{-1}$$

The steady-state excess carrier recombination rate

$$R' = g' = 2x10^{21}\ cm^{-3}s^{-1}$$

6.13

For $t < 0$, steady-state, so

$$\delta p(0) = g'\tau_{pO} = (5x10^{21})(3x10^{-7}) \Rightarrow$$

$$\delta p(0) = 1.5x10^{15}\ cm^{-3}$$

Now

$$\sigma = e\mu_n n_O + e(\mu_n + \mu_p)(\delta p)$$

For $t \ge 0$, $\delta p = \delta p(0)\exp(-t/\tau_{pO})$

Then

$$\sigma = (1.6x10^{-19})(1350)(5x10^{16})$$

$$+(1.6x10^{-19})(1350 + 480)(1.5x10^{15})\exp(-t/\tau_{pO})$$

or

$$\sigma = 10.8 + 0.439\exp(-t/\tau_{pO})$$

We have that

$$I = AJ = A\sigma E = \frac{A\sigma V}{L}$$

so

$$I = \frac{(10^{-4})(5)}{(0.10)}\left[10.8 + 0.439\exp(-t/\tau_{pO})\right]$$

or

$$I = \left[54 + 2.20\exp(-t/\tau_{pO})\right]\ mA$$

where

$$\tau_{pO} = 3x10^{-7}\ s$$

6.14

(a) p-type GaAs,

$$D_n\nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform generation rate, so that

$\nabla(\delta n) = \nabla^2(\delta n) = 0$, then

$$g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

The solution is of the form

$$\delta n = g'\tau_{nO}\left[1 - \exp(-t/\tau_{nO})\right]$$

Now

$$R'_n = \frac{\delta n}{\tau_{nO}} = g'\left[1 - \exp(-t/\tau_{nO})\right]$$

(b)

Maximum value at steady-state, $n_O = 10^{14}\ cm^{-3}$

So

$$(\delta n)_O = g'\tau_{nO} \Rightarrow \tau_{nO} = \frac{(\delta n)_O}{g'} = \frac{10^{14}}{10^{20}}$$

or

$$\tau_{nO} = 10^{-6}\ s$$

(c)

Determine *t* at which

(i) $\delta n = (0.75)x10^{14}\ cm^{-3}$

We have

$$0.75x10^{14} = 10^{14}\left[1 - \exp(-t/\tau_{nO})\right]$$

which yields

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.75}\right) \Rightarrow \quad t = 1.39\ \mu s$$

(ii) $\delta n = 0.5x10^{14}\ cm^{-3}$

We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.5}\right) \Rightarrow \quad t = 0.693\ \mu s$$

(iii) $\delta n = 0.25x10^{14}\ cm^{-3}$

We find

$$t = \tau_{nO}\ln\left(\frac{1}{1 - 0.25}\right) \Rightarrow \quad t = 0.288\ \mu s$$

6.15

(a)

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{15}} 2.25x10^4\ cm^{-3}$$

Then

$$R_{pO} = \frac{p_O}{\tau_{pO}} \Rightarrow \tau_{pO} = \frac{p_O}{R_{pO}} = \frac{2.25x10^4}{10^{11}}$$

or

$$\tau_{pO} = 2.25x10^{-7}\ s$$

Now

$$R'_p = \frac{\delta p}{\tau_{pO}} = \frac{10^{14}}{2.25x10^{-7}} \Rightarrow$$

or

$$R'_p = 4.44x10^{20}\ cm^{-3}s^{-1}$$

Recombination rate increases by the factor

$$\frac{R'_p}{R_{pO}} = \frac{4.44x10^{20}}{10^{11}} \Rightarrow \frac{R'_p}{R_{pO}} = 4.44x10^9$$

(b)

From part (a), $\tau_{pO} = 2.25x10^{-7}\ s$

6.16

Silicon, n-type. For $0 \le t \le 10^{-7}\ s$

$$\delta p = g'\tau_{pO}\left[1-\exp\left(-t/\tau_{pO}\right)\right]$$
$$= \left(2x10^{20}\right)\left(10^{-7}\right)\left[1-\exp\left(-t/\tau_{pO}\right)\right]$$

or

$$\delta p = 2x10^{13}\left[1-\exp\left(-t/\tau_{pO}\right)\right]$$

At $t = 10^{-7}\ s$,

$$\delta p\left(10^{-7}\right) = 2x10^{13}\left[1-\exp(-1)\right]$$

or

$$\delta p\left(10^{-7}\right) = 1.26x10^{13}\ cm^{-3}$$

For $t > 10^{-7}\ s$,

$$\delta p = \left(1.26x10^{13}\right)\exp\left[\frac{-\left(t-10^{-7}\right)}{\tau_{pO}}\right]$$

where

$$\tau_{pO} = 10^{-7}\ s$$

6.17

(a) For $0 < t < 2x10^{-6}\ s$

$$\delta n = g'\tau_{nO}\left[1-\exp\left(-t/\tau_{nO}\right)\right]$$
$$= \left(10^{20}\right)\left(10^{-6}\right)\left[1-\exp\left[-t/\tau_{nO}\right]\right]$$

or

$$\delta n = 10^{14}\left[1-\exp\left(-t/\tau_{nO}\right)\right]$$

where $\tau_{nO} = 10^{-6}\ s$

At $t = 2x10^{-6}\ s$

$$\delta n(2\ \mu s) = \left(10^{14}\right)\left[1-\exp(-2/1)\right]$$

or

$$\delta n(2\ \mu s) = 0.865x10^{14}\ cm^{-3}$$

For $t > 2x10^{-6}\ s$

$$\delta n = 0.865x10^{14}\exp\left[\frac{-\left(t-2x10^{-6}\right)}{\tau_{nO}}\right]$$

(b) (i) At $t = 0$, $\delta n = 0$

(ii) At $t = 2x10^{-6}\ s$, $\delta n = 0.865x10^{14}\ cm^{-3}$

(iii) At $t \to \infty$, $\delta n = 0$

6.18

p-type, minority carriers are electrons

In steady-state, $\frac{\partial(\delta n)}{\partial t} = 0$, then

(a)

$$D_n\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp\left(-x/L_n\right) + B\exp\left(+x/L_n\right)$$

But $\delta n = 0$ as $x \to \infty$ so that $B \equiv 0$.

At $x = 0$, $\delta n = 10^{13}\ cm^{-3}$

Then

$$\delta n = 10^{13}\exp\left(-x/L_n\right)$$

Now

$$L_n = \sqrt{D_n\tau_{nO}}\ \text{, where } D_n = \mu_n\left(\frac{kT}{e}\right)$$

or

$$D_n = (0.0259)(1200) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)\left(5x10^{-7}\right)} \Rightarrow$$

or

$$L_n = 39.4\ \mu m$$

(b)

$$J_n = eD_n \frac{d(\delta n)}{dx} = \frac{eD_n(10^{13})}{(-L_n)}\exp(-x/L_n)$$

$$= \frac{-(1.6x10^{-19})(31.1)(10^{13})}{39.4x10^{-4}}\exp(-x/L_n)$$

or

$$\underline{J_n = -12.6\exp(-x/L_n) \quad mA/cm^2}$$

6.19

(a) p-type silicon, $p_{pO} = 10^{14}\ cm^{-3}$ and

$$n_{pO} = \frac{n_i^2}{p_{pO}} = \frac{(1.5x10^{10})^2}{10^{14}} = 2.25x10^6\ cm^{-3}$$

(b) Excess minority carrier concentration

$$\delta n = n_p - n_{pO}$$

At $x = 0$, $n_p = 0$ so that

$$\delta n(0) = 0 - n_{pO} = -2.25x10^6\ cm^{-3}$$

(c) For the one-dimensional case,

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0 \quad \text{where} \quad L_n^2 = D_n\tau_{nO}$$

The general solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

For $x \to \infty$, δn remains finite, so that $B = 0$.
Then the solution is

$$\underline{\delta n = -n_{pO}\exp(-x/L_n)}$$

6.20

p-type so electrons are the minority carriers

$$D_n\nabla^2(\delta n) + \mu_n \mathrm{E} \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

For steady state, $\frac{\partial(\delta n)}{\partial t} = 0$ and for $x>0$,
$g' = 0$, $\mathrm{E} = 0$, so we have

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \quad \text{or} \quad \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

where $L_n^2 = D_n\tau_{nO}$

The solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

The excess concentration δn must remain finite, so that $B = 0$. At $x = 0$, $\delta n(0) = 10^{15}\ cm^{-3}$, so the solution is

$$\delta n = 10^{15}\exp(-x/L_n)$$

We have that $\mu_n = 1050\ cm^2/V-s$, then

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (1050)(0.0259) = 27.2\ cm^2/s$$

Then

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(27.2)(8x10^{-7})} \Rightarrow$$

$$L_n = 46.6\ \mu m$$

(a)

Electron diffusion current density at $x = 0$

$$J_n = eD_n \frac{d(\delta n)}{dx}\Big|_{x=0}$$

$$= eD_n \frac{d}{dx}\left[10^{15}\exp(-x/L_n)\right]_{x=0}$$

$$= \frac{-eD_n(10^{15})}{L_n} = \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}}$$

or

$$\underline{J_n = -0.934\ A/cm^2}$$

Since $\delta p = \delta n$, excess holes diffuse at the same rate as excess electrons, then

$$\underline{J_p(x=0) = +0.934\ A/cm^2}$$

(b)

At $x = L_n$,

$$J_n = eD_n \frac{d(\delta n)}{dx}\Big|_{x=L_n} = \frac{eD_n(10^{15})}{(-L_n)}\exp(-1)$$

$$= \frac{-(1.6x10^{-19})(27.2)(10^{15})}{46.6x10^{-4}}\exp(-1)$$

or

$$\underline{J_n = -0.344\ A/cm^2}$$

Then

$$\underline{J_p = +0.344\ A/cm^2}$$

6.21

n-type, so we have

$$D_p \frac{d^2(\delta p)}{dx^2} - \mu_p \mathrm{E}_O \frac{d(\delta p)}{dx} - \frac{\delta p}{\tau_{pO}} = 0$$

Assume the solution is of the form

$$\delta p = A\exp(sx)$$

Then

$$\frac{d(\delta p)}{dx} = As\exp(sx), \quad \frac{d^2(\delta p)}{dx^2} = As^2\exp(sx)$$

Substituting into the differential equation

$$D_p As^2\exp(sx) - \mu_p E_O As\exp(sx) - \frac{A\exp(sx)}{\tau_{pO}} = 0$$

or

$$D_p s^2 - \mu_p E_O s - \frac{1}{\tau_{pO}} = 0$$

Dividing by D_p

$$s^2 - \frac{\mu_p}{D_p}E_O s - \frac{1}{L_p^2} = 0$$

The solution for s is

$$s = \frac{1}{2}\left[\frac{\mu_p}{D_p}E_O \pm \sqrt{\left(\frac{\mu_p}{D_p}E_O\right)^2 + \frac{4}{L_p^2}}\right]$$

This can be rewritten as

$$s = \frac{1}{L_p}\left[\frac{\mu_p L_p E_O}{2D_p} \pm \sqrt{\left(\frac{\mu_p L_p E_O}{2D_p}\right)^2 + 1}\right]$$

We may define

$$\beta \equiv \frac{\mu_p L_p E_O}{2D_p}$$

Then

$$s = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

In order that $\delta p = 0$ for $x > 0$, use the minus sign for $x > 0$ and the plus sign for $x < 0$. Then the solution is

$$\delta p(x) = A\exp(s_- x) \text{ for } x > 0$$

$$\delta p(x) = A\exp(s_+ x) \text{ for } x < 0$$

where

$$s_\pm = \frac{1}{L_p}\left[\beta \pm \sqrt{1+\beta^2}\right]$$

6.22

Computer Plot

6.23

(a) From Equation [6.55],

$$D_n\frac{d^2(\delta n)}{dx^2} + \mu_n E_O\frac{d(\delta n)}{dx} - \frac{\delta n}{\tau_{nO}} = 0$$

or

$$\frac{d^2(\delta n)}{dx^2} + \frac{\mu_n}{D_n}E_O\frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

We have that

$$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) \text{ so we can define}$$

$$\frac{\mu_n}{D_n}E_O = \frac{E_O}{(kT/e)} \equiv \frac{1}{L'}$$

Then we can write

$$\frac{d^2(\delta n)}{dx^2} + \frac{1}{L'}\cdot\frac{d(\delta n)}{dx} - \frac{\delta n}{L_n^2} = 0$$

Solution will be of the form

$$\delta n = \delta n(0)\exp(-\alpha x) \quad \text{where } \alpha > 0$$

Then

$$\frac{d(\delta n)}{dx} = -\alpha(\delta n) \text{ and } \frac{d^2(\delta n)}{dx^2} = \alpha^2(\delta n)$$

Substituting into the differential equation, we have

$$\alpha^2(\delta n) + \frac{1}{L'}\cdot[-\alpha(\delta n)] - \frac{\delta n}{L_n^2} = 0$$

or

$$\alpha^2 - \frac{\alpha}{L'} - \frac{1}{L_n^2} = 0$$

which yields

$$\alpha = \frac{1}{L_n}\left\{\frac{L_n}{2L'} + \sqrt{\left(\frac{L_n}{2L'}\right)^2 + 1}\right\}$$

Note that if $E_O = 0$, $L' \to \infty$, then $\alpha = \frac{1}{L_n}$

(b)

$$L_n = \sqrt{D_n\tau_{nO}} \quad \text{where } D_n = \mu_n\left(\frac{dT}{e}\right)$$

or

$$D_n = (1200)(0.0259) = 31.1\ cm^2/s$$

Then

$$L_n = \sqrt{(31.1)(5x10^{-7})} = 39.4\ \mu m$$

For $E_O = 12\ V/cm$, then

$$L' = \frac{(kT/e)}{E_O} = \frac{0.0259}{12} = 21.6x10^{-4}\ cm$$

Then

$$\alpha = 5.75x10^{2}\ cm^{-1}$$

(c)
Force on the electrons due to the electric field is in the negative x-direction. Therefore, the effective diffusion of the electrons is reduced and the concentration drops off faster with the applied electric field.

6.24
p-type so the minority carriers are electrons, then

$$D_n \nabla^2(\delta n) + \mu_n E \bullet \nabla(\delta n) + g' - \frac{\delta n}{\tau_{nO}} = \frac{\partial(\delta n)}{\partial t}$$

Uniform illumination means that
$\nabla(\delta n) = \nabla^2(\delta n) = 0$. For $\tau_{nO} = \infty$, we are left with

$$\frac{d(\delta n)}{dt} = g' \quad \text{which gives} \quad \delta n = g't + C_1$$

For $t < 0$, $\delta n = 0$ which means that $C_1 = 0$. Then

$$\underline{\delta n = G'_O t} \quad \text{for} \quad 0 \le t \le T$$

For $t > T$, $g' = 0$ so we have $\frac{d(\delta n)}{dt} = 0$

Or

$$\underline{\delta n = G'_O T} \quad \text{(No recombination)}$$

6.25
n-type so minority carriers are holes, then

$$D_p \nabla^2(\delta p) - \mu_p E \bullet \nabla(\delta p) + g' - \frac{\delta p}{\tau_{pO}} = \frac{\partial(\delta p)}{\partial t}$$

We have $\tau_{pO} = \infty$, $E = 0$, $\frac{\partial(\delta p)}{\partial t} = 0$ (steady state). Then we have

$$D_p \frac{d^2(\delta p)}{dx^2} + g' = 0 \quad \text{or} \quad \frac{d^2(\delta p)}{dx^2} = -\frac{g'}{D_p}$$

For $-L < x < +L$, $g' = G'_O$ = constant. Then

$$\frac{d(\delta p)}{dx} = -\frac{G'_O}{D_p} x + C_1 \quad \text{and}$$

$$\delta p = -\frac{G'_O}{2D_p} x^2 + C_1 x + C_2$$

For $L < x < 3L$, $g' = 0$ so we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \ \text{so that}\ \frac{d(\delta p)}{dx} = C_3 \ \text{and}$$

$$\delta p = C_3 x + C_4$$

For $-3L < x < -L$, $g' = 0$ so that

$$\frac{d^2(\delta p)}{dx^2} = 0,\ \frac{d(\delta p)}{dx} = C_5,\ \text{and}$$

$$\delta p = C_5 x + C_6$$

The boundary conditions are
(1) $\delta p = 0$ at $x = +3L$; (2) $\delta p = 0$ at $x = -3L$;
(3) δp continuous at $x = +L$; (4) δp continuous at $x = -L$; The flux must be continuous so that
(5) $\frac{d(\delta p)}{dx}$ continuous at $x = +L$; (6) $\frac{d(\delta p)}{dx}$ continuous at $x = -L$.

Applying these boundary conditions, we find

$$\delta p = \frac{G'_O}{2D_p}(5L^2 - x^2) \quad \text{for} \quad -L < x < +L$$

$$\delta p = \frac{G'_O L}{D_p}(3L - x) \quad \text{for} \quad L < x < 3L$$

$$\delta p = \frac{G'_O L}{D_p}(3L + x) \quad \text{for} \quad -3L < x < -L$$

6.26

$$\mu_p = \frac{d}{E_O t} = \frac{0.75}{\left(\frac{2.5}{1}\right)(160x10^{-6})} = 1875\ cm^2/V-s$$

Then

$$D_p = \frac{(\mu_p E_O)^2 (\Delta t)^2}{16 t_O}$$

$$= \frac{\left[(1875)\left(\frac{2.5}{1}\right)\right]^2 (75.5x10^{-6})^2}{16(160x10^{-6})}$$

which gives

$$D_p = 48.9\ cm^2/s$$

From the Einstein relation,

$$\frac{D_p}{\mu_p} = \frac{kT}{e} = \frac{48.9}{1875} = 0.02608\ V$$

6.27

Assume that $f(x,t) = (4\pi Dt)^{-1/2} \exp\left(\frac{-x^2}{4Dt}\right)$ is the solution to the differential equation

$$D\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f}{\partial t}$$

To prove: we can write

$$\frac{\partial f}{\partial x} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

and

$$\frac{\partial^2 f}{\partial x^2} = (4\pi Dt)^{-1/2}\left(\frac{-2x}{4Dt}\right)^2 \exp\left(\frac{-x^2}{4Dt}\right) + (4\pi Dt)^{-1/2}\left(\frac{-2}{4Dt}\right)\exp\left(\frac{-x^2}{4Dt}\right)$$

Also

$$\frac{\partial f}{\partial t} = (4\pi Dt)^{-1/2}\left(\frac{-x^2}{4D}\right)\left(\frac{-1}{t^2}\right)\exp\left(\frac{-x^2}{4Dt}\right) + (4\pi D)^{-1/2}\left(\frac{-1}{2}\right)t^{-3/2}\exp\left(\frac{-x^2}{4Dt}\right)$$

Substituting the expressions for $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial f}{\partial t}$ into the differential equation, we find $0 = 0$, Q.E.D.

6.28

Computer Plot

6.29

n-type

$$\delta n = \delta p = g'\tau_{pO} = (10^{21})(10^{-6}) = 10^{15}\ cm^{-3}$$

We have $n_O = 10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Now

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}+10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.3498\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right) = (0.0259)\ln\left(\frac{2.25x10^4 + 10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.2877\ eV}$$

6.30

(a) p-type

$$E_{Fi} - E_F = kT\ln\left(\frac{p_O}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.3294\ eV}$$

(b)

$$\delta n = \delta p = 5x10^{14}\ cm^{-3}$$

and

$$n_O = \frac{(1.5x10^{10})^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$E_{Fn} - E_{Fi} = kT\ln\left(\frac{n_O + \delta n}{n_i}\right) = (0.0259)\ln\left(\frac{4.5x10^4 + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.2697\ eV}$$

and

$$E_{Fi} - E_{Fp} = kT\ln\left(\frac{p_O + \delta p}{n_i}\right) = (0.0259)\ln\left(\frac{5x10^{15} + 5x10^{14}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.3318\ eV}$$

6.31

n-type GaAs; $n_O = 5x10^{16}\ cm^{-3}$

$$p_O = \frac{n_i^2}{n_O} = \frac{\left(1.8x10^6\right)^2}{5x10^{16}} = 6.48x10^{-5}\ cm^{-3}$$

We have

$$\delta n = \delta p = (0.1)N_d = 5x10^{15}\ cm^{-3}$$

(a)

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_O + \delta n}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16} + 5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.6253\ eV}$$

We have

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{16}}{1.8x10^6}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.6228\ eV}$$

Now

$$E_{Fn} - E_F = \left(E_{Fn} - E_{Fi}\right) - \left(E_F - E_{Fi}\right)$$
$$= 0.6253 - 0.6228$$

so

$$\underline{E_{Fn} - E_F = 0.0025\ eV}$$

(b)

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_O + \delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{15}}{1.8x10^6}\right)$$

or

$$\underline{E_{Fi} - E_{Fp} = 0.5632\ eV}$$

6.32

Quasi-Fermi level for minority carrier electrons

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_O + \delta n}{n_i}\right)$$

We have

$$\delta n = \left(10^{14}\right)\left(\frac{x}{50\ \mu m}\right)$$

Neglecting the minority carrier electron concentration

$$E_{Fn} - E_{Fi} = kT \ln\left[\frac{\left(10^{14}\right)(x)}{(50\ \mu m)\left(1.8x10^6\right)}\right]$$

We find

$x(\mu m)$	$E_{Fn} - E_{Fi}$ (eV)
0	–0.581
1	+0.361
2	+0.379
10	+0.420
20	+0.438
50	+0.462

Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_O + \delta p}{n_i}\right)$$

We have $p_O = 10^{16}\ cm^{-3}$, $\delta p = \delta n$

We find

$x(\mu m)$	$E_{Fi} - E_{Fp}$ (eV)
0	+0.58115
50	+0.58140

6.33

(a) We can write

$$E_{Fi} - E_F = kT \ln\left(\frac{p_O}{n_i}\right)$$

and

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_O + \delta p}{n_i}\right)$$

so that

$$\left(E_{Fi} - E_{Fp}\right) - \left(E_{Fi} - E_F\right) = E_F - E_{Fp}$$

$$= kT \ln\left(\frac{p_O + \delta p}{n_i}\right) - kT \ln\left(\frac{p_O}{n_i}\right)$$

or

$$E_F - E_{Fp} = kT \ln\left(\frac{p_O + \delta p}{p_O}\right) = (0.01)kT$$

Then

$$\frac{p_O + \delta p}{p_O} = \exp(0.01) = 1.010 \Rightarrow$$

$$\frac{\delta p}{p_O} = 0.010 \Rightarrow \text{ low-injection, so that}$$

$$\underline{\delta p = 5x10^{12}\ cm^{-3}}$$

(b)

$$E_{Fn} - E_{Fi} \approx kT \ln\left(\frac{\delta p}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{12}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fn} - E_{Fi} = 0.1505\ eV}$$

6.34
Computer Plot

6.35
Computer Plot

6.36
(a)

$$R = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n\left(n + n'\right) + C_p\left(p + p'\right)}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}\left(n + n'\right) + \tau_{nO}\left(p + p'\right)}$$

For $n = p = 0$

$$R = \frac{-n_i^2}{\tau_{pO} n_i + \tau_{nO} n_i} \Rightarrow \underline{R = \frac{-n_i}{\tau_{pO} + \tau_{nO}}}$$

(b)
We had defined the net generation rate as
$g - R = g_O + g' - \left(R_O + R'\right)$ where
$g_O = R_O$ since these are the thermal equilibrium generation and recombination rates. If $g' = 0$, then $g - R = -R'$ and $R' = \frac{-n_i}{\tau_{pO} + \tau_{nO}}$ so that

$g - R = +\frac{n_i}{\tau_{pO} + \tau_{nO}}$. Thus a negative recombination rate implies a net positive generation rate.

6.37
We have that

$$R = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n\left(n + n'\right) + C_p\left(p + p'\right)}$$

$$= \frac{\left(np - n_i^2\right)}{\tau_{pO}\left(n + n_i\right) + \tau_{nO}\left(p + n_i\right)}$$

If $n = n_O + \delta n$ and $p = p_O + \delta n$, then

$$R = \frac{\left(n_O + \delta n\right)\left(p_O + \delta n\right) - n_i^2}{\tau_{pO}\left(n_O + \delta n + n_i\right) + \tau_{nO}\left(p_O + \delta n + n_i\right)}$$

$$= \frac{n_O p_O + \delta n\left(n_O + p_O\right) + (\delta n)^2 - n_i^2}{\tau_{pO}\left(n_O + \delta n + n_i\right) + \tau_{nO}\left(p_O + \delta n + n_i\right)}$$

If $\delta n << n_i$, we can neglect the $(\delta n)^2$; also
$n_O p_O = n_i^2$
Then

$$R = \frac{\delta n\left(n_O + p_O\right)}{\tau_{pO}\left(n_O + n_i\right) + \tau_{nO}\left(p_O + n_i\right)}$$

(a)
For n-type, $n_O >> p_O$, $n_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{pO}} = 10^{+7}\ s^{-1}}$$

(b)
Intrinsic, $n_O = p_O = n_i$
Then

$$\frac{R}{\delta n} = \frac{2n_i}{\tau_{pO}\left(2n_i\right) + \tau_{nO}\left(2n_i\right)}$$

or

$$\frac{R}{\delta n} = \frac{1}{\tau_{pO} + \tau_{nO}} = \frac{1}{10^{-7} + 5x10^{-7}} \Rightarrow$$

$$\underline{\frac{R}{\delta n} = 1.67x10^{+6}\ s^{-1}}$$

(c)
p-type, $p_O >> n_O$, $p_O >> n_i$
Then

$$\underline{\frac{R}{\delta n} = \frac{1}{\tau_{nO}} = \frac{1}{5x10^{-7}} = 2x10^{+6}\ s^{-1}}$$

6.38

(a) From Equation [6.56],

$$D_p \frac{d^2(\delta p)}{dx^2} + g' - \frac{\delta p}{\tau_{pO}} = 0$$

Solution is of the form

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p) + B\exp(+x/L_p)$$

At $x = \infty$, $\delta p = g'\tau_{pO}$, so that $B \equiv 0$,

Then

$$\delta p = g'\tau_{pO} + A\exp(-x/L_p)$$

We have

$$D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = s(\delta p)\Big|_{x=0}$$

We can write

$$\frac{d(\delta p)}{dx}\Big|_{x=0} = \frac{-A}{L_p} \text{ and } (\delta p)\Big|_{x=0} = g'\tau_{pO} + A$$

Then

$$\frac{-AD_p}{L_p} = s(g'\tau_{pO} + A)$$

Solving for A we find

$$A = \frac{-sg'\tau_{pO}}{\dfrac{D_p}{L_p} + s}$$

The excess concentration is then

$$\delta p = g'\tau_{pO}\left[1 - \frac{s}{(D_p/L_p) + s} \cdot \exp\left(\frac{-x}{L_p}\right)\right]$$

where

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(10)(10^{-7})} = 10^{-3} \ cm$$

Now

$$\delta p = (10^{21})(10^{-7})\left[1 - \frac{s}{(10/10^{-3}) + s}\exp\left(\frac{-x}{L_p}\right)\right]$$

or

$$\delta p = 10^{14}\left[1 - \frac{s}{10^4 + s}\exp\left(\frac{-x}{L_p}\right)\right]$$

(i) $s = 0$, $\delta p = 10^{14} \ cm^{-3}$

(ii) $s = 2000 \ cm/s$,

$$\delta p = 10^{14}\left[1 - 0.167\exp\left(\frac{-x}{L_p}\right)\right]$$

(iii) $s = \infty$, $\delta p = 10^{14}\left[1 - \exp\left(\frac{-x}{L_p}\right)\right]$

(b) (i) $s = 0$, $\delta p(0) = 10^{14} \ cm^{-3}$

(ii) $s = 2000 \ cm/s$, $\delta p(0) = 0.833x10^{14} \ cm^{-3}$

(iii) $s = \infty$, $\delta p(0) = 0$

6.39

$$L_n = \sqrt{D_n \tau_{nO}} = \sqrt{(25)(5x10^{-7})} = 35.4x10^{-4} \ cm$$

(a)

At $x = 0$, $g'\tau_{nO} = (2x10^{21})(5x10^{-7}) = 10^{15} \ cm^{-3}$

Or $\delta n_O = g'\tau_{nO} = 10^{15} \ cm^{-3}$

For $x > 0$

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{nO}} = 0 \Rightarrow \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Solution is of the form

$$\delta n = A\exp(-x/L_n) + B\exp(+x/L_n)$$

At $x = 0$, $\delta n = \delta n_O = A + B$

At $x = W$,

$$\delta n = 0 = A\exp(-W/L_n) + B\exp(+W/L_n)$$

Solving these two equations, we find

$$A = \frac{-\delta n_O \exp(+2W/L_n)}{1 - \exp(2W/L_n)}$$

$$B = \frac{\delta n_O}{1 - \exp(2W/L_n)}$$

Substituting into the general solution, we find

$$\delta n = \frac{\delta n_O}{[\exp(+W/L_n) - \exp(-W/L_n)]} \times \{\exp[+(W-x)/L_n] - \exp[-(W-x)/L_n]\}$$

or

$$\delta n = \frac{\delta n_O \sinh[(W-x)/L_n]}{\sinh[W/L_n]}$$

where

$\delta n_O = 10^{15} \ cm^{-3}$ and $L_n = 35.4 \ \mu m$

(b)

If $\tau_{nO} = \infty$, we have

$$\frac{d^2(\delta n)}{dx^2} = 0$$

so the solution is of the form

$$\delta n = Cx + D$$

Applying the boundary conditions, we find

$$\delta n = \delta n_O \left(1 - \frac{x}{W}\right)$$

6.40

For $\tau_{pO} = \infty$, we have

$$\frac{d^2(\delta p)}{dx^2} = 0 \text{ so that } \frac{d(\delta p)}{dx} = A \text{ and}$$

$$\delta p = Ax + B$$

At $x = W$

$$-D_p \frac{d(\delta p)}{dx}\Big|_{x=W} = s \cdot (\delta p)\Big|_{x=W}$$

or

$$-D_p A = s(AW + B)$$

which yields

$$B = \frac{-A}{s}\left(D_p + sW\right)$$

At $x = 0$, the flux of excess holes is

$$10^{19} = -D_p \frac{d(\delta p)}{dx}\Big|_{x=0} = -D_p A$$

so that

$$A = \frac{-10^{19}}{10} = -10^{18} \; cm^{-4}$$

and

$$B = \frac{10^{18}}{s}(10 + sW) = 10^{18}\left(\frac{10}{s} + W\right)$$

The solution is now

$$\delta p = 10^{18}\left(W - x + \frac{10}{s}\right)$$

(a)

For $s = \infty$,

$$\delta p = 10^{18}\left(20x10^{-4} - x\right) cm^{-3}$$

(b)

For $s = 2x10^3 \; cm/s$

$$\delta p = 10^{18}\left(70x10^{-4} - x\right) cm^{-3}$$

6.41

For $-W < x < 0$,

$$D_n \frac{d^2(\delta n)}{dx^2} + G_O' = 0$$

so that

$$\frac{d(\delta n)}{dx} = -\frac{G_O'}{D_p} x + C_1$$

and

$$\delta n = -\frac{G_O'}{2D_n} x^2 + C_1 x + C_2$$

For $0 < x < W$,

$$\frac{d^2(\delta n)}{dx^2} = 0 \text{, so } \frac{d(\delta n)}{dx} = C_3 \text{, } \delta n = C_3 x + C_4$$

The boundary conditions are:

(1) $s = 0$ at $x = -W$, so that $\frac{d(\delta n)}{dx}\Big|_{x=-W} = 0$

(2) $s = \infty$ at $x = +W$, so that $\delta n(W) = 0$

(3) δn continuous at $x = 0$

(4) $\frac{d(\delta n)}{dx}$ continuous at $x = 0$

Applying the boundary conditions, we find

$$C_1 = C_3 = -\frac{G_O' W}{D_n} \;,\; C_2 = C_4 = +\frac{G_O' W^2}{D_n}$$

Then, for $-W < x < 0$

$$\delta n = \frac{G_O'}{2D_n}\left(-x^2 - 2Wx + 2W^2\right)$$

and for $0 < x < +W$

$$\delta n = \frac{G_O' W}{D_n}(W - x)$$

6.42

Computer Plot

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