Chapter 7

Problem Solutions

7.1

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

where $V_t = 0.0259\ V$ and $n_i = 1.5x10^{10}\ cm^{-3}$

We find

(a)

For $N_d = 10^{15}\ cm^{-3}$	$V_{bi}(V)$
(i) $N_a = 10^{15}\ cm^{-3}$	$0.575\ V$
(ii) $N_a = 10^{16}\ cm^{-3}$	0.635
(iii) $N_a = 10^{17}\ cm^{-3}$	0.695
(iv) $N_a = 10^{18}\ cm^{-3}$	0.754

(b)

For $N_d = 10^{18}\ cm^{-3}$	$V_{bi}(V)$
(i) $N_a = 10^{15}\ cm^{-3}$	$0.754\ V$
(ii) $N_a = 10^{16}\ cm^{-3}$	0.814
(iii) $N_a = 10^{17}\ cm^{-3}$	0.874
(iv) $N_a = 10^{18}\ cm^{-3}$	0.933

7.2

Si: $n_i = 1.5x10^{10}\ cm^{-3}$

Ge: $n_i = 2.4x10^{13}\ cm^{-3}$

GaAs: $n_i = 1.8x10^{6}\ cm^{-3}$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \text{ and } V_t = 0.0259\ V$$

(a)

$N_d = 10^{14}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.635\ V}$, $\underline{Ge: V_{bi} = 0.253\ V}$,

$\underline{GaAs: V_{bi} = 1.10\ V}$

(b)

$N_d = 5x10^{16}\ cm^{-3}$, $N_a = 5x10^{16}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.778\ V}$, $\underline{Ge: V_{bi} = 0.396\ V}$,

$\underline{GaAs: V_{bi} = 1.25\ V}$

(c)

$N_d = 10^{17}\ cm^{-3}$, $N_a = 10^{17}\ cm^{-3}$

Then

$\underline{Si: V_{bi} = 0.814\ V}$, $\underline{Ge: V_{bi} = 0.432\ V}$,

$\underline{GaAs: V_{bi} = 1.28\ V}$

7.3

Computer Plot

7.4

Computer Plot

7.5

(a) n-side:

$$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3294\ eV}$$

p-side:

$$E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5x10^{10}}\right)$$

or

$$\underline{E_{Fi} - E_F = 0.4070\ eV}$$

(b)

$$V_{bi} = 0.3294 + 0.4070$$

or

$$\underline{V_{bi} = 0.7364\ V}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259) \ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.7363\ V}$$

(d)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}} \times\left(\frac{10^{17}}{5x10^{15}}\right)\left(\frac{1}{10^{17}+5x10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 0.426\ \mu m$$

Now

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.736)}{1.6x10^{-19}} \times\left(\frac{5x10^{15}}{10^{17}}\right)\left(\frac{1}{10^{17}+5x10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 0.0213\ \mu m$$

We have

$$|E_{max}| = \frac{eN_d x_n}{\in} = \frac{(1.6x10^{-19})(5x10^{15})(0.426x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|E_{max}| = 3.29x10^4\ V/cm$$

7.6

(a) n-side

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_F - E_{Fi} = 0.3653\ eV$$

p-side

$$E_{Fi} - E_F = (0.0259)\ln\left(\frac{2x10^{16}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_{Fi} - E_F = 0.3653\ eV$$

(b)

$$V_{bi} = 0.3653 + 0.3653 \Rightarrow$$

$$V_{bi} = 0.7306\ V$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.7305\ V$$

(d)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.7305)}{1.6x10^{-19}} \times\left(\frac{2x10^{16}}{2x10^{16}}\right)\left(\frac{1}{2x10^{16}+2x10^{16}}\right)\right]^{1/2}$$

or

$$x_n = 0.154\ \mu m$$

By symmetry

$$x_p = 0.154\ \mu m$$

Now

$$|E_{max}| = \frac{eN_d x_n}{\in} = \frac{(1.6x10^{-19})(2x10^{16})(0.154x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|E_{max}| = 4.76x10^4\ V/cm$$

7.7

(b) $$n_O = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right] = 2.8x10^{19}\exp\left(\frac{-0.21}{0.0259}\right)$$

or

$$n_O = N_d = 8.43x10^{15}\ cm^{-3}$$ (n-region)

$$p_O = N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right] = 1.04x10^{19}\exp\left(\frac{-0.18}{0.0259}\right)$$

or

$$p_O = N_a = 9.97x10^{15}\ cm^{-3}$$ (p-region)

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(9.97x10^{15})(8.43x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.690\ V}$$

7.8

(a) GaAs: $V_{bi} = 1.20\ V$, $n_i = 1.8x10^6\ cm^{-3}$

$$x_p = 0.2W = 0.2(x_n + x_p)$$

or

$$\frac{x_p}{x_n} = 0.25$$

Also

$$N_d x_n = N_a x_p \Rightarrow \frac{x_p}{x_n} = \frac{N_d}{N_a} = 0.25$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

or

$$1.20 = (0.0259)\ln\left(\frac{0.25N_a^2}{n_i^2}\right)$$

Then

$$\frac{0.25N_a^2}{n_i^2} = \exp\left(\frac{1.20}{0.0259}\right)$$

or

$$N_a = 2n_i \exp\left[\frac{1.20}{2(0.0259)}\right]$$

or

$$\underline{N_a = 4.14x10^{16}\ cm^{-3}}$$

(b)

$$N_d = 0.25N_a \Rightarrow \underline{N_d = 1.04x10^{16}\ cm^{-3}}$$

(c)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.20)}{1.6x10^{-19}} \times\left(\frac{4}{1}\right)\left(\frac{1}{4.14x10^{16} + 1.04x10^{16}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 0.366\ \mu m}$$

(d)

$$x_p = 0.25x_n \Rightarrow \underline{x_p = 0.0916\ \mu m}$$

(e)

$$E_{max} = \frac{eN_d x_n}{\in} = \frac{eN_a x_p}{\in}$$

$$= \frac{(1.6x10^{-19})(1.04x10^{16})(0.366x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{E_{max} = 5.25x10^4\ V/cm}$$

7.9

(a)

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.635\ V}$$

(b)

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{15}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_n = 0.864\ \mu m}$

Now

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.635)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{16} + 10^{15}}\right)\right]^{1/2}$$

or $\underline{x_p = 0.0864\ \mu m}$

(c)

$$E_{max} = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(0.864x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{E_{max} = 1.34x10^4\ V/cm}$$

7.10

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \text{ and}$$

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

We can write

$$N_C N_V = N_{CO} N_{VO} \left(\frac{T}{300}\right)^3$$

Now

$$V_{bi} = V_t\left[\ln(N_a N_d) - \ln(n_i^2)\right]$$

$$= V_t\left[\ln(N_a N_d) - \ln(N_{CO} N_{VO}) - \ln\left(\frac{T}{300}\right)^3 + \frac{E_g}{kT}\right]$$

or

$$V_{bi} = V_t\left[\ln\left(\frac{N_a N_d}{N_{CO} N_{VO}}\right) - 3\ln\left(\frac{T}{300}\right) + \frac{E_g}{kT}\right]$$

or

$$0.40 = (0.0250)\left(\frac{T}{300}\right) \times\left[\ln\left[\frac{(5x10^{15})(10^{16})}{(2.8x10^{19})(1.04x10^{19})}\right] - 3\ln\left(\frac{T}{300}\right) + \frac{1.12}{(0.0259)(T/300)}\right]$$

Then

$$15.44 = \left(\frac{T}{300}\right)\left[-15.58 - 3\ln\left(\frac{T}{300}\right) + \frac{43.24}{(T/300)}\right]$$

By trial and error

$$\underline{T = 490K}$$

7.11

(a) $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.8556\,V}$$

(b)
For a 1% change in V_{bi}, assume that the change is due to n_i^2, where the major dependence on temperature is given by

$$n_i^2 \propto \exp\left(\frac{-E_g}{kT}\right)$$

Now

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{\ln\left[\frac{N_a N_d}{n_i^2(T_2)}\right]}{\ln\left[\frac{N_a N_d}{n_i^2(T_1)}\right]}$$

$$= \frac{\ln(N_a N_d) - \ln\left[n_i^2(T_2)\right]}{\ln(N_a N_d) - \ln\left[n_i^2(T_1)\right]}$$

$$= \frac{\ln(N_a N_d) - \ln(N_C N_V) - \left(\frac{-E_g}{kT_2}\right)}{\ln(N_a N_d) - \ln(N_C N_V) - \left(\frac{-E_g}{kT_1}\right)}$$

$$= \left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_2}\right\}$$

$$/\left\{\ln\left[(5x10^{17})(10^{17})\right] - \ln\left[(2.8x10^{19})(1.04x10^{19})\right] + \frac{E_g}{kT_1}\right\}$$

or

$$\frac{V_{bi}(T_2)}{V_{bi}(T_1)} = \frac{79.897 - 88.567 + \frac{E_g}{kT_2}}{79.897 - 88.567 + \frac{E_g}{kT_1}}$$

We can write

$$0.990 = \frac{-8.67 + \frac{E_g}{kT_2}}{-8.67 + \frac{1.12}{0.0259}} = \frac{-8.67 + \frac{E_g}{kT_2}}{34.57}$$

so that

$$\frac{E_g}{kT_2} = 42.90 = \frac{1.12}{(0.0259)\left(\frac{T_2}{300}\right)}$$

We then find

$$\underline{T_2 = 302.4K}$$

7.12

(b) For $N_d = 10^{16}\ cm^{-3}$,

$$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.3473\ eV}$$

For $N_d = 10^{15}\ cm^{-3}$,

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

Then

$$V_{bi} = 0.3473 - 0.2877$$

or

$$\underline{V_{bi} = 0.0596\ V}$$

7.13

(a) $$V_{bi} = V_t\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{16})(10^{12})}{(1.5x10^{10})^2}\right]$$

or

$$\underline{V_{bi} = 0.456\ V}$$

(b)

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times\left(\frac{10^{12}}{10^{16}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_n = 2.43x10^{-7}\ cm}$$

(c)

$$x_p = \left[\frac{2(11.7)(8.85x10^{-14})(0.456)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{12}}\right)\left(\frac{1}{10^{16}+10^{12}}\right)\right]^{1/2}$$

or

$$\underline{x_p = 2.43x10^{-3}\ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(2.43x10^{-7})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.75x10^2\ V/cm}$$

7.14

Assume Silicon, so

$$L_D = \left(\frac{\in kT}{e^2 N_d}\right)^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)(1.6x10^{-19})}{(1.6x10^{-19})^2 N_d}\right]^{1/2}$$

or

$$L_D = \left(\frac{1.676x10^5}{N_d}\right)^{1/2}$$

(a) $N_d = 8x10^{14}\ cm^{-3}$, $\underline{L_D = 0.1447\ \mu m}$

(b) $N_d = 2.2x10^{16}\ cm^{-3}$, $\underline{L_D = 0.02760\ \mu m}$

(c) $N_d = 8x10^{17}\ cm^{-3}$, $\underline{L_D = 0.004577\ \mu m}$

Now

(a) $V_{bi} = 0.7427\ V$

(b) $V_{bi} = 0.8286\ V$

(c) $V_{bi} = 0.9216\ V$

Also

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi})}{1.6x10^{-19}} \times\left(\frac{8x10^{17}}{N_d}\right)\left(\frac{1}{8x10^{17}+N_d}\right)\right]^{1/2}$$

Then

(a) $\underline{x_n = 1.096\ \mu m}$

(b) $\underline{x_n = 0.2178\ \mu m}$

(c) $\underline{x_n = 0.02731\ \mu m}$

Now

(a) $\underline{\dfrac{L_D}{x_n} = 0.1320}$

(b) $\dfrac{L_D}{x_n} = 0.1267$

(c) $\dfrac{L_D}{x_n} = 0.1677$

7.15
Computer Plot

7.16

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(2x10^{16})(2x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.671\ V$

(b)

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times\left[\frac{2x10^{16}+2x10^{15}}{(2x10^{16})(2x10^{15})}\right]\right\}^{1/2}$$

or

$$W = \left[7.12x10^{-9}(V_{bi}+V_R)\right]^{1/2}$$

For $V_R = 0$, $W = 0.691x10^{-4}\ cm$

For $V_R = 8\ V$, $W = 2.48x10^{-4}\ cm$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W}$$

For $V_R = 0$, $E_{max} = 1.94x10^4\ V/cm$

For $V_R = 8\ V$, $E_{max} = 7.0x10^4\ V/cm$

7.17

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{17})(10^{17})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.856\ V$

(b)

$$x_n = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times\left(\frac{5x10^{17}}{1x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$x_n = 0.251\ \mu m$

Also

$$x_p = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(5.856)}{1.6x10^{-19}} \times\left(\frac{1x10^{17}}{5x10^{17}}\right)\left(\frac{1}{5x10^{17}+1x10^{17}}\right)\right]^{1/2}$$

or

$x_p = 0.0503\ \mu m$

Also

$W = x_n + x_p$

or

$W = 0.301\ \mu m$

(c)

$$E_{max} = \frac{2(V_{bi}+V_R)}{W} = \frac{2(5.856)}{0.301x10^{-4}}$$

or

$E_{max} = 3.89x10^5\ V/cm$

(d)

$$C_T = \frac{\in A}{W} = \frac{(11.7)(8.85x10^{-14})(10^{-4})}{0.301x10^{-4}}$$

or

$C_T = 3.44\ pF$

7.18

(a) $V_{bi} = V_t \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{50N_a^2}{(1.5x10^{10})^2}\right]$$

We can write

$$\exp\left(\frac{0.752}{0.0259}\right) = \frac{50N_a^2}{\left(1.5x10^{10}\right)^2}$$

or

$$N_a = \frac{1.5x10^{10}}{\sqrt{50}}\exp\left[\frac{0.752}{2(0.0259)}\right]$$

and

$$\underline{N_a = 4.28x10^{15}\ cm^{-3}}$$

Then

$$\underline{N_d = 2.14x10^{17}\ cm^{-3}}$$

(b)

$$x_p \approx W \approx \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\cdot\left(\frac{1}{N_a}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(10.752)}{\left(1.6x10^{-19}\right)\left(4.28x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_p = 1.80\ \mu m}$$

(c)

$$C' \approx \left[\frac{e\in N_a}{2\left(V_{bi}+V_R\right)}\right]^{1/2}$$

$$= \left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(4.28x10^{15}\right)}{2(10.752)}\right]^{1/2}$$

or

$$\underline{C' = 5.74x10^{-9}\ F/cm^2}$$

7.19

(a) Neglecting change in V_{bi}

$$\frac{C'(2N_a)}{C'(N_a)} = \left\{\frac{\left[\dfrac{2}{(2N_a+N_d)}\right]}{\left(\dfrac{1}{N_a+N_d}\right)}\right\}^{1/2}$$

For a $n^+p \Rightarrow N_d \gg N_a$

Then

$$\frac{C'(2N_a)}{C'(N_a)} = \sqrt{2} = 1.414$$

so a <u>41.4% change</u>.

(b)

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln\left(\dfrac{2N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

$$= \frac{kT\ln 2 + kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}{kT\ln\left(\dfrac{N_aN_d}{n_i^2}\right)}$$

So we can write this as

$$\frac{V_{bi}(2N_a)}{V_{bi}(N_a)} = \frac{kT\ln 2 + V_{bi}(N_a)}{V_{bi}(N_a)}$$

so

$$\Delta V_{bi} = kT\ln 2 = (0.0259)\ln 2$$

or

$$\underline{\Delta V_{bi} = 17.95\ mV}$$

17.20

(a)

$$\frac{W(A)}{W(B)} = \frac{\left[\dfrac{2\in\left(V_{biA}+V_R\right)}{e}\left(\dfrac{N_a+N_{dA}}{N_aN_{dA}}\right)\right]^{1/2}}{\left[\dfrac{2\in\left(V_{biB}+V_R\right)}{e}\left(\dfrac{N_a+N_{dB}}{N_aN_{dB}}\right)\right]^{1/2}}$$

or

$$\frac{W(A)}{W(B)} = \left[\frac{\left(V_{biA}+V_R\right)}{\left(V_{biB}+V_R\right)}\cdot\frac{\left(N_a+N_{dA}\right)}{\left(N_a+N_{dB}\right)}\cdot\left(\frac{N_{dB}}{N_{dA}}\right)\right]^{1/2}$$

We find

$$V_{biA} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.7543\ V$$

$$V_{biB} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.8139\ V$$

So we find

$$\frac{W(A)}{W(B)} = \left[\left(\frac{5.7543}{5.8139}\right)\left(\frac{10^{18}+10^{15}}{10^{18}+10^{16}}\right)\left(\frac{10^{16}}{10^{15}}\right)\right]^{1/2}$$

or

$$\underline{\frac{W(A)}{W(B)} = 3.13}$$

(b)

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)} = \frac{\dfrac{2(V_{biA}+V_R)}{W(A)}}{\dfrac{2(V_{biB}+V_R)}{W(B)}} = \frac{W(B)}{W(A)} \cdot \frac{(V_{biA}+V_R)}{(V_{biB}+V_R)}$$

$$= \left(\frac{1}{3.13}\right)\left(\frac{5.7543}{5.8139}\right)$$

or

$$\frac{\mathrm{E}(A)}{\mathrm{E}(B)} = 0.316$$

(c)

$$\frac{C_j'(A)}{C_j'(B)} = \frac{\left[\dfrac{\in N_a N_{dA}}{2(V_{biA}+V_R)(N_a+N_{dA})}\right]^{1/2}}{\left[\dfrac{\in N_a N_{dB}}{2(V_{biB}+V_R)(N_a+N_{dB})}\right]^{1/2}}$$

$$= \left[\left(\frac{N_{dA}}{N_{dB}}\right)\left(\frac{V_{biB}+V_R}{V_{biA}+V_R}\right)\left(\frac{N_a+N_{dB}}{N_a+N_{dA}}\right)\right]^{1/2}$$

$$= \left[\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{5.8139}{5.7543}\right)\left(\frac{10^{18}+10^{16}}{10^{18}+10^{15}}\right)\right]^{1/2}$$

or

$$\frac{C_j'(A)}{C_j'(B)} = 0.319$$

17.21

(a) $V_{bi} = (0.0259)\ln\left[\dfrac{(4x10^{15})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$

$$V_{bi} = 0.766\ V$$

Now

$$|\mathrm{E}_{\max}| = \left[\frac{2e(V_{bi}+V_R)}{\in}\left(\frac{N_a N_d}{N_a+N_d}\right)\right]^{1/2}$$

so

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times \left[\frac{(4x10^{15})(4x10^{17})}{4x10^{15}+4x10^{17}}\right]$$

or

$$9x10^{10} = 1.22x10^9 (V_{bi}+V_R) \Rightarrow$$

$$V_{bi}+V_R = 73.77\ V$$

and

$$V_R = 73\ V$$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(4x10^{16})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$$

$$V_{bi} = 0.826\ V$$

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times \left[\frac{(4x10^{16})(4x10^{17})}{4x10^{16}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R = 8.007\ V$$

and

$$V_R = 7.18\ V$$

(c)

$$V_{bi} = (0.0259)\ln\left[\frac{(4x10^{17})(4x10^{17})}{(1.5x10^{10})^2}\right] \Rightarrow$$

$$V_{bi} = 0.886\ V$$

$$(3x10^5)^2 = \left[\frac{2(1.6x10^{-19})}{(11.7)(8.85x10^{-14})}\right](V_{bi}+V_R) \times \left[\frac{(4x10^{17})(4x10^{17})}{4x10^{17}+4x10^{17}}\right]$$

which yields

$$V_{bi}+V_R = 1.456\ V$$

and

$$V_R = 0.570\ V$$

17.22

(a) We have

$$\frac{C_j(0)}{C_j(10)} = \frac{\left[\dfrac{\in N_a N_d}{2V_{bi}(N_a+N_d)}\right]^{1/2}}{\left[\dfrac{\in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}}$$

or

$$\frac{C_j(0)}{C_j(10)} = 3.13 = \left(\frac{V_{bi}+V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10\,V$, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10$$

or

$$\underline{V_{bi} = 1.14\,V}$$

(b)

$$x_p = 0.2W = 0.2(x_p + x_n)$$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow$$

so

$$1.14 = (0.0259)\ln\left[\frac{0.25N_a^2}{(1.8x10^6)^2}\right]$$

We can then write

$$N_a = \frac{1.8x10^6}{\sqrt{0.25}}\exp\left[\frac{1.14}{2(0.0259)}\right]$$

or

$$\underline{N_a = 1.3x10^{16}\ cm^{-3}}$$

and

$$\underline{N_d = 3.25x10^{15}\ cm^{-3}}$$

7.23

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(5x10^{16})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.20\,V$$

Now

$$\frac{C_j'(V_{R1})}{C_j'(V_{R2})} = \frac{\left[\frac{1}{V_{bi}+V_{R1}}\right]^{1/2}}{\left[\frac{1}{V_{bi}+V_{R2}}\right]^{1/2}} = \left[\frac{V_{bi}+V_{R2}}{V_{bi}+V_{R1}}\right]^{1/2}$$

So

$$(3)^2 = \frac{1.20 + V_{R2}}{1.20 + 1} \Rightarrow$$

$$\underline{V_{R2} = 18.6\,V}$$

7.24

$$C' = \left[\frac{e \in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.754\,V$$

For $N_a >> N_d$, we have

$$C' = \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(10^{15})}{2(V_{bi}+V_R)}\right]^{1/2}$$

or

$$C' = \left[\frac{8.28x10^{-17}}{V_{bi}+V_R}\right]^{1/2}$$

For $V_R = 1\,V$, $C' = 6.87x10^{-9}\ F/cm^2$

For $V_R = 10\,V$, $C' = 2.77x10^{-9}\ F/cm^2$

If $A = 6x10^{-4}\ cm^2$, then

For $V_R = 1\,V$, $C = 4.12\ pF$

For $V_R = 10\,V$, $C = 1.66\ pF$

The resonant frequency is given by

$$f_O = \frac{1}{2\pi\sqrt{LC}}$$

so that

For $V_R = 1\,V$, $\underline{f_O = 1.67\ MHz}$

For $V_R = 10\,V$, $\underline{f_O = 2.63\ MHz}$

7.25

$$|E_{max}| = \frac{eN_d x_n}{\in}$$

For a p^+n junction,

$$x_n \approx \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

so that

$$|E_{max}| = \left[\frac{2eN_d}{\in}(V_{bi}+V_R)\right]^{1/2}$$

Assuming that $V_{bi} << V_R$, then

$$N_d = \frac{\in E_{max}^2}{2eV_R} = \frac{(11.7)(8.85x10^{-14})(10^6)^2}{2(1.6x10^{-19})(10)}$$

or

$$N_d = 3.24x10^{17} \ cm^{-3}$$

7.26

$$x_n = 0.1W = 0.1(x_n + x_p)$$

which yields

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} = 9$$

We can write

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{9N_a^2}{(1.5x10^{10})^2}\right]$$

We also have

$$C'_j = \frac{C_T}{A} = \frac{3.5x10^{-12}}{5.5x10^{-4}} = 6.36x10^{-9} \ F/cm^2$$

so

$$6.36x10^{-9} = \left[\frac{e \in N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$$

Which becomes

$$4.05x10^{-17} = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_a(9N_a)}{2(V_{bi}+V_R)(N_a+9N_a)}$$

or

$$4.05x10^{-17} = \frac{7.46x10^{-32} N_a}{(V_{bi}+V_R)}$$

If $V_R = 1.2 \ V$, then by iteration we find

$$N_a = 9.92x10^{14} \ cm^{-3}$$

$$V_{bi} = 0.632 \ V$$

$$N_d = 8.93x10^{15} \ cm^{-3}$$

7.27

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(10^{14})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.557 \ V$$

(b)

$$x_p = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{10^{14}}{5x10^{15}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$x_p = 5.32x10^{-6} \ cm$$

Also

$$x_n = \left[\frac{2\in V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.557)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 2.66x10^{-4} \ cm$$

(c)

For $x_n = 30 \ \mu m$, we have

$$30x10^{-4} = \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times \left(\frac{5x10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5x10^{15}}\right)\right]^{1/2}$$

which becomes

$$9x10^{-6} = 1.27x10^{-7}(V_{bi}+V_R)$$

We find

$$V_R = 70.3 \ V$$

7.28

An n^+p junction with $N_a = 10^{14} \ cm^{-3}$,

(a)

A one-sided junction and assume $V_R >> V_{bi}$, then

$$x_p = \left[\frac{2\in V_R}{eN_a}\right]^{1/2}$$

so

$$\left(50x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{14}\right)}$$

which yields

$$\underline{V_R = 193\ V}$$

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p\left(\frac{N_a}{N_d}\right)$$

so

$$x_n = \left(50x10^{-4}\right)\left(\frac{10^{14}}{10^{16}}\right) \Rightarrow$$

or

$$\underline{x_n = 0.5\ \mu m}$$

(c)

$$E_{max} = \frac{2\left(V_{bi}+V_R\right)}{W} = \frac{2(193)}{50.5x10^{-4}}$$

or

$$\underline{E_{max} = 7.72x10^4\ V/cm}$$

7.29

(a) $$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.796\ V$$

$$C = AC' = A\left[\frac{e\in N_a N_d}{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}\right]^{1/2}$$

$$= \left(5x10^{-5}\right)\left[\frac{\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)}{2\left(V_{bi}+V_R\right)} \times \frac{\left(10^{18}\right)\left(5x10^{15}\right)}{\left(10^{18}+5x10^{15}\right)}\right]^{1/2}$$

or

$$C = \left(5x10^{-5}\right)\left[\frac{4.121x10^{-16}}{\left(V_{bi}+V_R\right)}\right]^{1/2}$$

For $V_R = 0$, $\underline{C = 1.14\ pF}$

For $V_R = 3\ V$, $\underline{C = 0.521\ pF}$

For $V_R = 6\ V$, $\underline{C = 0.389\ pF}$

We can write

$$\left(\frac{1}{C}\right)^2 = \frac{1}{A^2}\left[\frac{2\left(V_{bi}+V_R\right)\left(N_a+N_d\right)}{e\in N_a N_d}\right]$$

For the p^+n junction

$$\left(\frac{1}{C}\right)^2 \approx \frac{1}{A^2}\left[\frac{2\left(V_{bi}+V_R\right)}{e\in N_d}\right]$$

so that

$$\frac{\Delta(1/C)^2}{\Delta V_R} = \frac{1}{A^2}\cdot\frac{2}{e\in N_d}$$

We have

For $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23}$

For $V_R = 6\ V$, $\left(\frac{1}{C}\right)^2 = 6.61x10^{24}$

Then, for $\Delta V_R = 6\ V$,

$$\Delta(1/C)^2 = 5.84x10^{24}$$

We find

$$N_d = \frac{2}{A^2 e\in}\cdot\frac{1}{\left(\frac{\Delta(1/C)^2}{\Delta V_R}\right)}$$

$$= \frac{2}{\left(5x10^{-5}\right)^2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)} \times \frac{1}{\left(\frac{5.84x10^{24}}{6}\right)}$$

so that

$$\underline{N_d = 4.96x10^{15} \approx 5x10^{15}\ cm^{-3}}$$

Now, for a straight line

$$y = mx + b$$

$$m = \frac{\Delta(1/C)^2}{\Delta V_R} = \frac{5.84x10^{24}}{6}$$

At $V_R = 0$, $\left(\frac{1}{C}\right)^2 = 7.69x10^{23} = b$

Then

$$\left(\frac{1}{C}\right)^2 = \left(\frac{5.84x10^{24}}{6}\right)\cdot V_R + 7.69x10^{23}$$

Now, at $\left(\frac{1}{C}\right)^2 = 0$,

$$0 = \left(\frac{5.84x10^{24}}{6}\right)\cdot V_R + 7.69x10^{23}$$

which yields

$$\underline{V_R = -V_{bi} = -0.790\ V}$$

or

$$V_{bi} \approx 0.796\ V$$

(b)

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{18})(6x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.860\ V$$

$$C = (5x10^{-5})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi}+V_R)} \times \frac{(10^{18})(6x10^{16})}{(10^{18}+6x10^{16})}\right]^{1/2}$$

or

$$C = (5x10^{-5})\left[\frac{4.689x10^{-15}}{V_{bi}+V_R}\right]^{1/2}$$

Then

For $V_R = 0$, $\underline{C = 3.69\ pF}$

For $V_R = 3\ V$, $\underline{C = 1.74\ pF}$

For $V_R = 6\ V$, $\underline{C = 1.31\ pF}$

7.30

$$C' = \frac{C}{A} = \frac{1.3x10^{-12}}{10^{-5}} = 1.3x10^{-7}\ F/cm^2$$

(a) For a one-sided junction

$$C' = \left[\frac{e \in N_L}{2(V_{bi}+V_R)}\right]^{1/2}$$

where N_L is the doping concentration in the low-doped region.

We have $V_{bi} + V_R = 0.95 + 0.05 = 1.00\ V$

Then

$$(1.3x10^{-7})^2 = \frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})N_L}{2(1)}$$

which yields

$$\underline{N_L = 2.04x10^{17}\ cm^{-3}}$$

(b)

$$V_{bi} = V_t \ln\left(\frac{N_L N_H}{n_i^2}\right)$$

where N_H is the doping concentration in the high-doped region.

So

$$0.95 = (0.0259)\ln\left[\frac{(2.04x10^{17})N_H}{(1.5x10^{10})^2}\right]$$

which yields

$$\underline{N_H = 9.38x10^{18}\ cm^{-3}}$$

7.31

Computer Plot

7.32

(a) $V_{bi} = V_t \ln\left(\frac{N_{aO}N_{dO}}{n_i^2}\right)$

(c) p-region

$$\frac{dE}{dx} = \frac{\rho(x)}{\in} = \frac{-eN_{aO}}{\in}$$

or

$$E = \frac{-eN_{aO}x}{\in} + C_1$$

We have

$$E = 0 \text{ at } x = -x_p \Rightarrow C_1 = \frac{-eN_{aO}x_p}{\in}$$

Then for $\underline{-x_p < x < 0}$

$$\underline{E = \frac{-eN_{aO}}{\in}(x + x_p)}$$

n-region, $0 < x < x_O$

$$\frac{dE_1}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{2\in}$$

or

$$E_1 = \frac{eN_{dO}x}{2\in} + C_2$$

n-region, $x_O < x < x_n$

$$\frac{dE_2}{dx} = \frac{\rho(x)}{\in} = \frac{eN_{dO}}{\in}$$

or

$$E_2 = \frac{eN_{dO}x}{\in} + C_3$$

We have $E_2 = 0$ at $x = x_n$, then

$$C_3 = \frac{-eN_{dO}x_n}{\in}$$

so that for $\underline{x_O < x < x_n}$

$$\underline{E_2 = \frac{-eN_{dO}}{\in}(x_n - x)}$$

We also have

$E_2 = E_1$ at $x = x_O$

Then

$$\frac{eN_{dO}x_O}{2\in} + C_2 = \frac{-eN_{dO}}{\in}\left(x_n - x_O\right)$$

or

$$C_2 = \frac{-eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)$$

Then, for $0 < x < x_O$,

$$E_1 = \frac{eN_{dO}x}{2\in} - \frac{eN_{dO}}{\in}\left(x_n - \frac{x_O}{2}\right)$$

7.33

(a) $$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\in} = \frac{-dE(x)}{dx}$$

For $-2 < x < -1\ \mu m$, $\rho(x) = +eN_d$

So

$$\frac{dE}{dx} = \frac{eN_d}{\in} \Rightarrow E = \frac{eN_d x}{\in} + C_1$$

At $x = -2\ \mu m \equiv -x_O$, $E = 0$

So

$$0 = \frac{-eN_d x_O}{\in} + C_1 \Rightarrow C_1 = \frac{eN_d x_O}{\in}$$

Then

$$E = \frac{eN_d}{\in}\left(x + x_O\right)$$

At $x = 0$, $E(0) = E(x = -1\ \mu m)$, so

$$E(0) = \frac{eN_d}{\in}(-1+2)x10^{-4}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{(11.7)\left(8.85x10^{-14}\right)}\left(1x10^{-4}\right)$$

which yields

$$E(0) = 7.73x10^4\ V/cm$$

(c)

Magnitude of potential difference is

$$|\phi| = \int E dx = \frac{eN_d}{\in}\int\left(x + x_O\right)dx$$

$$= \frac{eN_d}{\in}\left(\frac{x^2}{2} + x_O \cdot x\right) + C_2$$

Let $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{eN_d}{\in}\left(\frac{x_O^2}{2} - x_O^2\right) + C_2 \Rightarrow C_2 = \frac{eN_d x_O^2}{2\in}$$

Then we can write

$$|\phi| = \frac{eN_d}{2\in}\left(x + x_O\right)^2$$

At $x = -1\ \mu m$

$$|\phi_1| = \frac{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}{2(11.7)\left(8.85x10^{-14}\right)}\left[(-1+2)x10^{-4}\right]$$

or

$$|\phi_1| = 3.86\ V$$

Potential difference across the intrinsic region

$$|\phi_i| = E(0)\cdot d = \left(7.73x10^4\right)\left(2x10^{-4}\right)$$

or

$$|\phi_i| = 15.5\ V$$

By symmetry, potential difference across the p-region space charge region is also $3.86\ V$. The total reverse-bias voltage is then

$$V_R = 2(3.86) + 15.5 \Rightarrow V_R = 23.2\ V$$

7.34

(a) For the linearly graded junction,

$\rho(x) = eax$,

Then

$$\frac{dE}{dx} = \frac{\rho(x)}{\in} = \frac{eax}{\in}$$

Now

$$E = \int \frac{eax}{\in}dx = \frac{ea}{\in}\cdot\frac{x^2}{2} + C_1$$

At $x = +x_O$ and $x = -x_O$, $E = 0$

So

$$0 = \frac{ea}{\in}\left(\frac{x_O^2}{2}\right) + C_1 \Rightarrow C_1 = \frac{-ea}{\in}\left(\frac{x_O^2}{2}\right)$$

Then

$$E = \frac{ea}{2\in}\left(x^2 - x_O^2\right)$$

(b)

$$\phi(x) = -\int E dx = \frac{-ea}{2\in}\left[\frac{x^3}{3} - x_O^2 \cdot x\right] + C_2$$

Set $\phi = 0$ at $x = -x_O$, then

$$0 = \frac{-ea}{2\in}\left[\frac{-x_O^3}{3} + x_O^3\right] + C_2 \Rightarrow C_2 = \frac{eax_O^3}{3\in}$$

Then

$$\phi(x) = \frac{-ea}{2\in}\left(\frac{x^3}{3} - x_O^2\cdot x\right) + \frac{eax_O^3}{3\in}$$

7.35

We have that

$$C' = \left[\frac{ea \in^2}{12(V_{bi} + V_R)} \right]^{1/3}$$

then

$$(7.2x10^{-9})^3$$
$$= \left[\frac{a(1.6x10^{-19})[(11.7)(8.85x10^{-14})]^2}{12(0.7 + 3.5)} \right]$$

which yields

$$\underline{a = 1.1x10^{20} \ cm^{-4}}$$