Chapter 8

Problem Solutions

8.1
In the forward bias
$$I_f \approx I_S \exp\left(\frac{eV}{kT}\right)$$
Then
$$\frac{I_{f1}}{I_{f2}} = \frac{I_S}{I_S}\cdot\frac{\exp\left(\frac{eV_1}{kT}\right)}{\exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\frac{e}{kT}(V_1 - V_2)\right]$$
or
$$V_1 - V_2 = \left(\frac{kT}{e}\right)\ln\left(\frac{I_{f1}}{I_{f2}}\right)$$
(a)
For $\frac{I_{f1}}{I_{f2}} = 10 \Rightarrow$ $\underline{V_1 - V_2 = 59.9\ mV \approx 60\ mV}$

(b)
For $\frac{I_{f1}}{I_{f2}} = 100 \Rightarrow V_1 - V_2 = 119.3\ mV \approx 120 mV$

8.2
$$I = I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$
or we can write this as
$$\frac{I}{I_S} + 1 = \exp\left(\frac{eV}{kT}\right)$$
so that
$$V = \left(\frac{kT}{e}\right)\ln\left(\frac{I}{I_S} + 1\right)$$
In reverse bias, I is negative, so at
$\frac{I}{I_S} = -0.90$, we have
$$V = (0.0259)\ln(1 - 0.90) \Rightarrow$$
or
$$V = -59.6\ mV$$

8.3
Computer Plot

8.4
The cross-sectional area is
$$A = \frac{I}{J} = \frac{10x10^{-3}}{20} = 5x10^{-4}\ cm^2$$
We have
$$J \approx J_S \exp\left(\frac{V_D}{V_t}\right) \Rightarrow\ 20 = J_S \exp\left(\frac{0.65}{0.0259}\right)$$
so that
$$J_S = 2.52x10^{-10}\ A/cm^2$$
We can write
$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_{pO}}}\right]\backslash$$
We want
$$\frac{\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}}}{\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_{pO}}}} = 0.10$$
or
$$\frac{\frac{1}{N_a}\cdot\sqrt{\frac{25}{5x10^{-7}}}}{\frac{1}{N_a}\cdot\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d}\cdot\sqrt{\frac{10}{5x10^{-7}}}}$$
$$= \frac{7.07x10^3}{7.07x10^3 + \frac{N_a}{N_d}\left(4.47x10^3\right)} = 0.10$$
which yields
$$\frac{N_a}{N_d} = 14.24$$
Now
$$J_S = 2.52x10^{-10} = \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2$$
$$\times\left[\frac{1}{(14.24)N_d}\cdot\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{N_d}\cdot\sqrt{\frac{10}{5x10^{-7}}}\right]$$
We find
$$N_d = 7.1x10^{14}\ cm^{-3}$$
and
$$\underline{N_a = 1.01x10^{16}\ cm^{-3}}$$

8.5
(a)

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}$$

$$=\frac{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}}{\sqrt{\dfrac{D_n}{\tau_{nO}}}\cdot\dfrac{n_i^2}{N_a}+\sqrt{\dfrac{D_p}{\tau_{pO}}}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{1}{1+\sqrt{\dfrac{D_p\tau_{nO}}{D_n\tau_{pO}}}\cdot\left(\dfrac{N_a}{N_d}\right)}$$

We have

$$\frac{D_p}{D_n}=\frac{\mu_p}{\mu_n}=\frac{1}{2.4}\quad\text{and}\quad\frac{\tau_{nO}}{\tau_{pO}}=\frac{1}{0.1}$$

so

$$\frac{J_n}{J_n+J_p}=\frac{1}{1+\sqrt{\dfrac{1}{2.4}\cdot\dfrac{1}{0.1}}\left(\dfrac{N_a}{N_d}\right)}$$

or

$$\frac{J_n}{J_n+J_p}=\frac{1}{1+(2.04)\left(\dfrac{N_a}{N_d}\right)}$$

(b)
Using Einstein's relation, we can write

$$\frac{J_n}{J_n+J_p}=\frac{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}}{\dfrac{e\mu_n}{L_n}\cdot\dfrac{n_i^2}{N_a}+\dfrac{e\mu_p}{L_p}\cdot\dfrac{n_i^2}{N_d}}$$

$$=\frac{e\mu_n N_d}{e\mu_n N_d+\dfrac{L_n}{L_p}\cdot e\mu_p N_a}$$

We have

$$\sigma_n=e\mu_n N_d\quad\text{and}\quad\sigma_p=e\mu_p N_a$$

Also

$$\frac{L_n}{L_p}=\sqrt{\frac{D_n\tau_{nO}}{D_p\tau_{pO}}}=\sqrt{\frac{2.4}{0.1}}=4.90$$

Then

$$\frac{J_n}{J_n+J_p}=\frac{(\sigma_n/\sigma_p)}{(\sigma_n/\sigma_p)+4.90}$$

8.6
For a silicon p^+n junction,

$$I_S=Aen_i^2\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$=\left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2\cdot\frac{1}{10^{16}}\sqrt{\frac{12}{10^{-7}}}$$

or

$$I_S=3.94x10^{-15}\ A$$

Then

$$I_D=I_S\exp\left(\frac{V_D}{V_t}\right)=\left(3.94x10^{-15}\right)\exp\left(\frac{0.50}{0.0259}\right)$$

or

$$I_D=9.54x10^{-7}\ A$$

8.7
We want

$$\frac{J_n}{J_n+J_p}=0.95$$

$$=\frac{\dfrac{eD_n n_{pO}}{L_n}}{\dfrac{eD_n n_{pO}}{L_n}+\dfrac{eD_p p_{nO}}{L_p}}=\frac{\dfrac{D_n}{L_n N_a}}{\dfrac{D_n}{L_n N_a}+\dfrac{D_p}{L_p N_d}}$$

$$=\frac{\dfrac{D_n}{L_n}}{\dfrac{D_n}{L_n}+\dfrac{D_p}{L_p}\cdot\dfrac{N_a}{N_d}}$$

We obtain

$$L_n=\sqrt{D_n\tau_{nO}}=\sqrt{(25)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_n=15.8\ \mu m$$

$$L_p=\sqrt{D_p\tau_{pO}}=\sqrt{(10)\left(0.1x10^{-6}\right)}\Rightarrow$$

$$L_p=10\ \mu m$$

Then

$$0.95 = \frac{\frac{25}{15.8}}{\frac{25}{15.8} + \frac{10}{10} \cdot \left(\frac{N_a}{N_d}\right)}$$

which yields

$$\frac{N_a}{N_d} = 0.083$$

8.8

(a) p-side: $E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{5x10^{15}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_{Fi} - E_F = 0.329\ eV$$

Also

n-side: $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{10^{17}}{1.5x10^{10}}\right) \Rightarrow$$

$$E_F - E_{Fi} = 0.407\ eV$$

(b)
We can find

$$D_n = (1250)(0.0259) = 32.4\ cm^2 / s$$

$$D_p = (420)(0.0259) = 10.9\ cm^2 / s$$

Now

$$J_S = en_i^2 \left[\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}} \right]$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2$$

$$\times \left[\frac{1}{5x10^{15}} \sqrt{\frac{32.4}{10^{-6}}} + \frac{1}{10^{17}} \sqrt{\frac{10.9}{10^{-7}}} \right]$$

or

$$J_S = 4.48x10^{-11}\ A / cm^2$$

Then

$$I_S = AJ_S = \left(10^{-4}\right)\left(4.48x10^{-11}\right)$$

or

$$I_S = 4.48x10^{-15}\ A$$

We find

$$I = I_S \exp\left(\frac{V_D}{V_t}\right)$$

$$= \left(4.48x10^{-15}\right) \exp\left(\frac{0.5}{0.0259}\right)$$

or

$$I = 1.08\ \mu A$$

(c)
The hole current is proportional to

$$I_P \propto en_i^2 \cdot A \cdot \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2 \left(10^{-4}\right)\left(\frac{1}{10^{17}}\right)\sqrt{\frac{10.9}{10^{-7}}}$$

or

$$I_P \propto 3.76x10^{-16}\ A$$

Then

$$\frac{I_P}{I} = \frac{3.76x10^{-16}}{4.48x10^{-15}} \Rightarrow \frac{I_P}{I} = 0.0839$$

8.9

$$I = I_S \left[\exp\left(\frac{V_a}{V_t}\right) - 1 \right]$$

For a p^+n diode,

$$I_S = A\left(\frac{eD_p p_{nO}}{L_p}\right) = A\left(e\sqrt{\frac{D_p}{\tau_{pO}}} \cdot \frac{n_i^2}{N_d}\right)$$

$$= \left(10^{-4}\right)\left[\left(1.6x10^{-19}\right)\sqrt{\frac{10}{10^{-6}}} \cdot \frac{\left(2.4x10^{13}\right)^2}{10^{16}}\right]$$

or

$$I_S = 2.91x10^{-9}\ A$$

(a)
For $V_a = +0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{0.2}{0.0259}\right) - 1\right]$$

or

$$I = 6.55\ \mu A$$

(b)
For $V_a = -0.2\ V$,

$$I = \left(2.91x10^{-9}\right)\left[\exp\left(\frac{-0.2}{0.0259}\right) - 1\right]$$

$$\approx -2.91x10^{-9}\ A$$

or $I = -I_S = -2.91\ nA$

8.10

For an n^+p silicon diode

$$I_S = Aen_i^2 \cdot \frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}}$$

$$= \frac{\left(10^{-4}\right)\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)^2}{10^{16}}\sqrt{\frac{25}{10^{-6}}}$$

or

$$\underline{I_S = 1.8x10^{-15}\ A}$$

(a)

For $V_a = 0.5\,V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = \left(1.8x10^{-15}\right)\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 4.36x10^{-7}\ A}$$

(b)

For $V_a = -0.5\,V$

$$\underline{I_D = -I_S = -1.8x10^{-15}\ A}$$

8.11

(a) We find

$$D_p = \mu_p\left(\frac{kT}{e}\right) = (480)(0.0259) = 12.4\ cm^2/s$$

and

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(12.4)\left(0.1x10^{-6}\right)} \Rightarrow$$

$$L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} = 2.25x10^5\ cm^{-3}$$

Then

$$J_{pO} = \frac{eD_p p_{nO}}{L_p} = \frac{\left(1.6x10^{-19}\right)(12.4)\left(2.25x10^5\right)}{\left(11.1x10^{-4}\right)}$$

or

$$J_{pO} = 4.02x10^{-10}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{pO} = 4.02x10^{-14}\ A}$$

(b)

We have

$$D_n = \mu_n\left(\frac{kT}{e}\right) = (1350)(0.0259) = 35\ cm^2/s$$

and

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(35)\left(0.4x10^{-6}\right)} \Rightarrow$$

$$L_n = 37.4\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} = \frac{\left(1.6x10^{-19}\right)(35)\left(4.5x10^4\right)}{\left(37.4x10^{-4}\right)}$$

or

$$J_{nO} = 6.74x10^{-11}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{nO} = 6.74x10^{-15}\ A}$$

(c)

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{\left(5x10^{15}\right)\left(10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.617\ V$$

Then for

$$V_a = \frac{1}{2}V_{bi} = 0.309\ V$$

We find

$$p_n = p_{nO}\exp\left(\frac{eV_a}{kT}\right)$$

$$= \left(2.25x10^5\right)\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{p_n = 3.42x10^{10}\ cm^{-3}}$$

(d)

The total current is

$$I = \left(I_{pO} + I_{nO}\right)\exp\left(\frac{eV_a}{kT}\right)$$

$$= \left(4.02x10^{-14} + 6.74x10^{-15}\right)\exp\left(\frac{0.309}{0.0259}\right)$$

or

$$\underline{I = 7.13x10^{-9}\ A}$$

The hole current is

$$I_p = I_{pO} \exp\left(\frac{eV_a}{kT}\right) \exp\left[\frac{-(x - x_n)}{L_p}\right]$$

The electron current is given by

$$I_n = I - I_p$$

$$= 7.13x10^{-9} - (4.02x10^{-14}) \times \exp\left(\frac{0.309}{0.0259}\right) \exp\left[\frac{-(x - x_n)}{L_p}\right]$$

At $x = x_n + \frac{1}{2} L_p$

$$I_n = 7.13x10^{-9} - (6.10x10^{-9}) \exp\left(\frac{-1}{2}\right)$$

or

$$\underline{I_n = 3.43x10^{-9} \ A}$$

8.12

(a) The excess hole concentration is given by

$$\delta p_n = p_n - p_{nO}$$

$$= p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{-x}{L_p}\right)$$

We find

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4 \ cm^{-3}$$

and

$$L_p = \sqrt{D_p \tau_{pO}} = \sqrt{(8)(0.01x10^{-6})} \Rightarrow$$

$$L_p = 2.83 \ \mu m$$

Then

$$\delta p_n = (2.25x10^4) \times\left[\exp\left(\frac{0.610}{0.0259}\right) - 1\right] \exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

or

$$\underline{\delta p_n = 3.81x10^{14} \exp\left(\frac{-x}{2.83x10^{-4}}\right) cm^{-3}}$$

(b)

We have

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}$$

$$= \frac{eD_p(3.81x10^{14})}{(2.83x10^{-4})} \exp\left(\frac{-x}{2.83x10^{-4}}\right)$$

At $x = 3x10^{-4} \ cm$,

$$J_p = \frac{(1.6x10^{-19})(8)(3.81x10^{14})}{2.83x10^{-4}} \exp\left(\frac{-3}{2.83}\right)$$

or

$$\underline{J_p = 0.597 \ A / cm^2}$$

(c)

We have

$$J_{nO} = \frac{eD_n n_{pO}}{L_n} \exp\left(\frac{eV_a}{kT}\right)$$

We can determine that

$$n_{pO} = 4.5x10^3 \ cm^{-3} \text{ and } L_n = 10.7 \ \mu m$$

Then

$$J_{nO} = \frac{(1.6x10^{-19})(23)(4.5x10^3)}{10.7x10^{-4}} \exp\left(\frac{0.610}{0.0259}\right)$$

or

$$J_{nO} = 0.262 \ A / cm^2$$

We can also find

$$\underline{J_{pO} = 1.72 \ A / cm^2}$$

Then, at $x = 3 \ \mu m$,

$$J_n(3 \ \mu m) = J_{nO} + J_{pO} - J_p(3 \ \mu m)$$

$$= 0.262 + 1.72 - 0.597$$

or

$$\underline{J_n(3 \ \mu m) = 1.39 \ A / cm^2}$$

8.13

(a) From Problem 8.9 (Ge diode)

Low injection means

$$p_n(0) = (0.1)N_d = 10^{15} \ cm^{-3}$$

Now

$$p_{nO} = \frac{n_i^2}{N_d} = \frac{(2.4x10^{13})^2}{10^{16}} = 5.76x10^{10} \ cm^{-3}$$

We have

$$p_n(0) = p_{nO} \exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left[\frac{p_n(0)}{p_{nO}}\right]$$

$$= (0.0259) \ln\left(\frac{10^{15}}{5.76x10^{10}}\right)$$

or

$$\underline{V_a = 0.253 \ V}$$

(b)

For Problem 8.10 (Si diode)

$$n_p(0) = (0.1)N_a = 10^{15}\ cm^{-3}$$

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$V_a = V_t \ln\left[\frac{n_p(0)}{n_{pO}}\right]$$

$$= (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_a = 0.635\ V}$$

8.14

The excess electron concentration is given by

$$\delta n_p = n_p - n_{pO}$$

$$= n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{-x}{L_n}\right)$$

The total number of excess electrons is

$$N_p = A\int_0^\infty \delta n_p dx$$

We may note that

$$\int_0^\infty \exp\left(\frac{-x}{L_n}\right)dx = -L_n \exp\left(\frac{-x}{L_n}\right)\Bigg|_0^\infty = L_n$$

Then

$$N_p = AL_n n_{pO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We can find

$$D_n = 35\ cm^2/s \quad \text{and} \quad L_n = 59.2\ \mu m$$

Also

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.5x10^{10})^2}{8x10^{15}} = 2.81x10^4\ cm^{-3}$$

Then

$$N_p = (10^{-3})(59.2x10^{-4})(2.81x10^4) \times\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

or

$$N_p = 0.166\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then we find the total number of excess electrons in the p-region to be:

(a) $V_a = 0.3\ V$, $\underline{N_p = 1.78x10^4}$

(b) $V_a = 0.4\ V$, $\underline{N_p = 8.46x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_p = 4.02x10^7}$

Similarly, the total number of excess holes in the n-region is found to be:

$$N_n = AL_p p_{nO}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

We find that

$$D_p = 12.4\ cm^2/s \quad \text{and} \quad L_p = 11.1\ \mu m$$

Also

$$p_{nO} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

Then

$$N_n = (2.50x10^{-2})\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

So

(a) $V_a = 0.3\ V$, $\underline{N_n = 2.68x10^3}$

(b) $V_a = 0.4\ V$, $\underline{N_n = 1.27x10^5}$

(c) $V_a = 0.5\ V$, $\underline{N_n = 6.05x10^6}$

8.15

$$I \propto n_i^2 \exp\left(\frac{eV_a}{kT}\right) \propto \exp\left(\frac{-E_g}{kT}\right)\exp\left(\frac{eV_a}{kT}\right)$$

Then

$$I \propto \exp\left(\frac{eV_a - E_g}{kT}\right)$$

so

$$\frac{I_1}{I_2} = \frac{\exp\left(\frac{eV_{a1} - E_{g1}}{kT}\right)}{\exp\left(\frac{eV_{a2} - E_{g2}}{kT}\right)}$$

or

$$\frac{I_1}{I_2} = \exp\left(\frac{eV_{a1} - eV_{a2} - E_{g1} + E_{g2}}{kT}\right)$$

We have

$$\frac{10x10^{-3}}{10x10^{-6}} = \exp\left(\frac{0.255 - 0.32 - 0.525 + E_{g2}}{0.0259}\right)$$

or

$$10^3 = \exp\left(\frac{E_{g2} - 0.59}{0.0259}\right)$$

Then

$$E_{g2} = 0.59 + (0.0259)\ln(10^3)$$

which yields

$$\underline{E_{g2} = 0.769\ eV}$$

8.16

(a) We have

$$I_S = Aen_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

which can be written in the form

$$I_S = C'n_i^2$$

$$= C'N_{CO}N_{VO}\left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

or

$$\underline{I_S = CT^3 \exp\left(\frac{-E_g}{kT}\right)}$$

(b)

Taking the ratio

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{T_2}{T_1}\right)^3 \cdot \frac{\exp\left(\frac{-E_g}{kT_2}\right)}{\exp\left(\frac{-E_g}{kT_1}\right)}$$

$$= \left(\frac{T_2}{T_1}\right)^3 \cdot \exp\left[+E_g\left(\frac{1}{kT_1} - \frac{1}{kT_2}\right)\right]$$

For $T_1 = 300K$, $kT_1 = 0.0259$, $\frac{1}{kT_1} = 38.61$

For $T_2 = 400K$, $kT_2 = 0.03453$, $\frac{1}{kT_2} = 28.96$

(i) Germanium, $E_g = 0.66\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \exp[(0.66)(38.61 - 28.96)]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1383}$$

(ii) Silicon, $E_g = 1.12\ eV$

$$\frac{I_{S2}}{I_{S1}} = \left(\frac{400}{300}\right)^3 \cdot \exp[(1.12)(38.61 - 28.96)]$$

or

$$\underline{\frac{I_{S2}}{I_{S1}} = 1.17x10^5}$$

8.17

Computer Plot

8.18

One condition:

$$\left|\frac{I_f}{I_r}\right| = \frac{J_S \exp\left(\frac{eV_a}{kT}\right)}{J_S} = \exp\left(\frac{eV_a}{kT}\right) = 10^4$$

or

$$\frac{kT}{e} = \frac{V_a}{\ln(10^4)} = \frac{0.5}{\ln(10^4)}$$

or

$$\frac{kT}{e} = 0.05429 = (0.0259)\left(\frac{T}{300}\right)$$

which yields

$$T = 629K$$

Second conditon:

$$I_S = A\left(\frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}\right)$$

$$= Aen_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$$

$$= AeN_C N_V\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]\exp\left(\frac{-E_g}{kT}\right)$$

which becomes

$$10^{-6} = (10^{-4})(1.6x10^{-19})(2.8x10^{19})(1.04x10^{19})$$

$$\times\left(\frac{1}{5x10^{18}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{10^{15}}\sqrt{\frac{10}{10^{-7}}}\right)\exp\left(\frac{-E_g}{kT}\right)$$

or

$$\exp\left(\frac{+E_g}{kT}\right) = 4.66x10^{10}$$

For $E_g = 1.10\ eV$,

$$kT = \frac{E_g}{\ln(4.66x10^{10})} = \frac{1.10}{\ln(4.66x10^{10})}$$

or

$$kT = 0.04478\ eV = (0.0259)\left(\frac{T}{300}\right)$$

Then

$$T = 519K$$

This second condition yields a smaller temperature, so the maximum temperature is

$$\underline{T = 519K}$$

8.19

(a) We can write for the n-region

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The general solution is

$$\delta p_n = A\exp(+x/L_p) + B\exp(-x/L_p)$$

The boundary condition at $x = x_n$ gives

$$\delta p_n(x_n) = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= A\exp(+x_n/L_p) + B\exp(-x_n/L_p)$$

and the boundary condition at $x = x_n + W_n$ gives

$$\delta p_n(x_n + W_n) = 0$$

$$= A\exp[(x_n + W_n)/L_p] + B\exp[-(x_n + W_n)/L_p]$$

From this equation, we have

$$A = -B\exp[-2(x_n + W_n)/L_p]$$

Then, from the first boundary condition, we obtain

$$p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

$$= B\exp[-(x_n + 2W_n)/L_p] + B\exp(-x_n/L_p)$$

$$= B\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]$$

We then obtain

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\exp(-x_n/L_p)[1 - \exp(-2W_n/L_p)]}$$

which can be written in the form

$$B = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp[(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

Also

$$A = \frac{-p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\exp[-(x_n + W_n)/L_p]}{\exp(W_n/L_p) - \exp(-W_n/L_p)}$$

The solution can now be written as

$$\delta p_n = \frac{p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{2\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left\{\exp\left[\frac{(x_n + W_n - x)}{L_p}\right] - \exp\left[\frac{-(x_n + W_n - x)}{L_p}\right]\right\}$$

or finally,

$$\delta p_n = p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]\cdot\frac{\sinh\left(\frac{x_n + W_n - x}{L_p}\right)}{\sinh\left(\frac{W_n}{L_p}\right)}$$

(b)

$$J_p = -eD_p\frac{d(\delta p_n)}{dx}\Big|_{x=x_n}$$

$$= \frac{-eD_p p_{nO}\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]}{\sinh\left(\frac{W_n}{L_p}\right)}$$

$$\times\left(\frac{-1}{L_p}\right)\cosh\left(\frac{x_n + W_n - x}{L_p}\right)\Big|_{x=x_n}$$

Then

$$J_p = \frac{eD_p p_{nO}}{L_p}\coth\left(\frac{W_n}{L_p}\right)\cdot\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

8.20

$$I_D \propto n_i^2\exp\left(\frac{V_D}{V_t}\right)$$

For the temperature range $300 \le T \le 320K$, neglect the change in N_C and N_V

So

$$I_D \propto \exp\left(\frac{-E_g}{kT}\right)\cdot\exp\left(\frac{eV_D}{kT}\right)$$

$$\propto \exp\left[\frac{-(E_g - eV_D)}{kT}\right]$$

Taking the ratio of currents, but maintaining I_D a constant, we have

$$1 = \frac{\exp\left[\dfrac{-(E_g - eV_{D1})}{kT_1}\right]}{\exp\left[\dfrac{-(E_g - eV_{D2})}{kT_2}\right]} \Rightarrow$$

$$\frac{E_g - eV_{D1}}{kT_1} = \frac{E_g - eV_{D2}}{kT_2}$$

We have

$T = 300K$, $V_{D1} = 0.60\ V$ and

$$kT_1 = 0.0259\ eV\ ,\ \frac{kT_1}{e} = 0.0259\ V$$

$T = 310K$

$$kT_2 = 0.02676\ eV\ ,\ \frac{kT_2}{e} = 0.02676\ V$$

$T = 320K$

$$kT_3 = 0.02763\ eV\ ,\ \frac{kT_3}{e} = 0.02763\ V$$

So, for $\underline{T = 310K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D2}}{0.02676}$$

which yields

$$\underline{V_{D2} = 0.5827\ V}$$

For $\underline{T = 320K}$,

$$\frac{1.12 - 0.60}{0.0259} = \frac{1.12 - V_{D3}}{0.02763}$$

which yields

$$\underline{V_{D3} = 0.5653\ V}$$

8.21
Computer Plot

8.22

$$g_d = \frac{e}{kT} \cdot I_D = \frac{2x10^{-3}}{0.0259}$$

or

$$\underline{g_d = 0.0772\ S}$$

Also

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)(I_{pO}\tau_{pO} + I_{nO}\tau_{nO})$$

We have

$$\tau_{pO} = \tau_{nO} = 10^{-6}\ s$$

$$I_{pO} + I_{nO} = 2x10^{-3}\ A$$

Then

$$C_d = \frac{(2x10^{-3})(10^{-6})}{2(0.0259)} \Rightarrow$$

$$\underline{C_d = 3.86x10^{-8}\ F}$$

Then

$$Y = g_d + j\omega C_d$$

or

$$\underline{Y = 0.0772 + j\omega(3.86x10^{-8})}$$

8.23 For a p^+n diode

$$g_d = \frac{I_{DQ}}{V_t}\ ,\quad C_d = \frac{I_{DQ}\tau_{pO}}{2V_t}$$

Now

$$g_d = \frac{10^{-3}}{0.0259} = 3.86x10^{-2}\ S$$

and

$$C_d = \frac{(10^{-3})(10^{-7})}{2(0.0259)} = 1.93x10^{-9}\ F$$

Now

$$Z = \frac{1}{Y} = \frac{1}{g_d + j\omega C_d} = \frac{g_d - j\omega C_d}{g_d^2 + \omega^2 C_d^2}$$

We have $\omega = 2\pi f$,

We find:

$f = 10\ kHz$: $\underline{Z = 25.9 - j0.0814}$

$f = 100\ kHz$: $\underline{Z = 25.9 - j0.814}$

$f = 1\ MHz$: $\underline{Z = 23.6 - j7.41}$

$f = 10\ MHz$: $\underline{Z = 2.38 - j7.49}$

8.24
(b)
Two capacitances will be equal at some forward-bias voltage.
For a forward-bias voltage, the junction capacitance is

$$C_j = A\left[\frac{e \in N_a N_d}{2(V_{bi} - V_a)(N_a + N_d)}\right]^{1/2}$$

The diffusion capacitance is

$$C_d = \left(\frac{1}{2V_t}\right)(I_{pO}\tau_{pO} + I_{nO}\tau_{nO})$$

where

$$I_{pO} = \frac{Aen_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

and

$$I_{nO} = \frac{Aen_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We find

$$D_p = (320)(0.0259) = 8.29\ cm^2/s$$

$$D_n = (850)(0.0259) = 22.0\ cm^2/s$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{17})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.7363\ V$$

Now, we obtain

$$C_j = (10^{-4})\left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})}{2(V_{bi} - V_a)} \times \frac{(5x10^{15})(10^{17})}{(5x10^{15} + 10^{17})}\right]^{1/2}$$

or

$$C_j = (10^{-4})\left[\frac{3.945x10^{-16}}{(V_{bi} - V_a)}\right]^{1/2}$$

We also obtain

$$I_{pO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{10^{17}}\sqrt{\frac{8.29}{10^{-7}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{pO} = 3.278x10^{-16}\exp\left(\frac{V_a}{V_t}\right)$$

Also

$$I_{nO} = \frac{(10^{-4})(1.6x10^{-19})(1.5x10^{10})^2}{5x10^{15}}\sqrt{\frac{22}{10^{-6}}} \times \exp\left(\frac{V_a}{V_t}\right)$$

or

$$I_{nO} = 3.377x10^{-15}\exp\left(\frac{V_a}{V_t}\right)$$

We can now write

$$C_d = \frac{1}{2(0.0259)}\left[(3.278x10^{-16})(10^{-7}) + (3.377x10^{-15})(10^{-6})\right] \cdot \exp\left(\frac{V_a}{V_t}\right)$$

or

$$C_d = 6.583x10^{-20} \cdot \exp\left(\frac{V_a}{V_t}\right)$$

We want to set $C_j = C_d$

So

$$(10^{-4})\left[\frac{3.945x10^{-16}}{0.7363 - V_a}\right]^{1/2} = 6.583x10^{-20} \cdot \exp\left(\frac{V_a}{0.0259}\right)$$

By trial and error, we find

$$\underline{V_a = 0.463\ V}$$

At this voltage,

$$\underline{C_j = C_d \approx 3.8\ pF}$$

8.25

For a p^+n diode, $I_{pO} >> I_{nO}$, then

$$C_d = \left(\frac{1}{2V_t}\right)(I_{pO}\tau_{pO})$$

Now

$$\frac{\tau_{pO}}{2V_t} = 2.5x10^{-6}\ F/A$$

Then

$$\tau_{pO} = 2(0.0259)(2.5x10^{-6})$$

or

$$\underline{\tau_{pO} = 1.3x10^{-7}\ s}$$

At $1\ mA$,

$$C_d = (2.5x10^{-6})(10^{-3}) \Rightarrow$$

$$\underline{C_d = 2.5x10^{-9}\ F}$$

8.26

(a) $C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{pO}\tau_{pO} + I_{nO}\tau_{nO})$

For a one-sided n^+p diode, $I_{nO} >> I_{pO}$, then

$$C_d = \frac{1}{2}\left(\frac{e}{kT}\right)A(I_{nO}\tau_{nO})$$

so

$$10^{-12} = \frac{1}{2}\left(\frac{1}{0.0259}\right)\left(10^{-3}\right)\left(I_{nO}\right)\left(10^{-7}\right)$$

or

$$\underline{I_{nO} = I_D = 0.518\ mA}$$

(b)

$$I_{nO} = A\frac{eD_n n_{pO}}{L_n}\exp\left(\frac{V_a}{V_t}\right)$$

We find

$$L_n = \sqrt{D_n \tau_{nO}} = 15.8\ \mu m \text{ and}$$

$$n_{pO} = \frac{n_i^2}{N_a} = 2.25x10^4\ cm^{-3}$$

Then

$$0.518x10^{-3} = \frac{\left(1.6x10^{-19}\right)(25)\left(2.25x10^4\right)\left(10^{-3}\right)}{15.8x10^{-4}}\exp\left(\frac{V_a}{V_t}\right)$$

or

$$0.518x10^{-3} = 5.70x10^{-14}\exp\left(\frac{V_a}{0.0259}\right)$$

We find

$$\underline{V_a = 0.594\ V}$$

(c)

$$g_d = \left(\frac{e}{kT}\right)I_D = \frac{1}{r_d} \Rightarrow$$

$$r_d = \frac{0.0259}{0.518x10^{-3}}$$

or

$$\underline{r_d = 50\ \Omega}$$

8.27

(a) p-region

$$R_p = \frac{\rho_p L}{A} = \frac{L}{\sigma_p A} = \frac{L}{A\left(e\mu_p N_a\right)}$$

so

$$R_p = \frac{0.2}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(480)\left(10^{16}\right)}$$

or

$$R_p = 26\ \Omega$$

n-region

$$R_n = \frac{\rho_n L}{A} = \frac{L}{\sigma_n A} = \frac{L}{A\left(e\mu_n N_d\right)}$$

so

$$R_n = \frac{0.10}{\left(10^{-2}\right)\left(1.6x10^{-19}\right)(1350)\left(10^{15}\right)}$$

or

$$R_n = 46.3\ \Omega$$

The total series resistance is

$$R = R_p + R_n = 26 + 46.3 \Rightarrow$$

$$\underline{R = 72.3\ \Omega}$$

(b)

$$V = IR \Rightarrow 0.1 = I(72.3)$$

or

$$\underline{I = 1.38\ mA}$$

8.28

$$R = \frac{\rho_n L(n)}{A(n)} + \frac{\rho_p L(p)}{A(p)}$$

$$= \frac{(0.2)\left(10^{-2}\right)}{2x10^{-5}} + \frac{(0.1)\left(10^{-2}\right)}{2x10^{-5}}$$

or

$$\underline{R = 150\ \Omega}$$

We can write

$$V = I_D R + V_t \ln\left(\frac{I_D}{I_S}\right)$$

(a) (i) $I_D = 1\ mA$

$$V = \left(10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right)$$

or

$$\underline{V = 0.567\ V}$$

(ii) $I_D = 10\ mA$

$$V = \left(10x10^{-3}\right)(150) + (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right)$$

or $\underline{V = 1.98\ V}$

(b)

For $R = 0$

(i) $I_D = 1\ mA$

$$V = (0.0259)\ln\left(\frac{10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.417\ V}$$

(ii) $I_D = 10\ mA$

$$V = (0.0259)\ln\left(\frac{10x10^{-3}}{10^{-10}}\right) \Rightarrow$$

$$\underline{V = 0.477\ V}$$

8.29

$$r_d = 48\ \Omega = \frac{1}{g_d} \Rightarrow g_d = 0.0208$$

We have

$$g_d = \frac{e}{kT}\cdot I_D \Rightarrow I_D = (0.0208)(0.0259)$$

or

$$\underline{I_D = 0.539\ mA}$$

Also

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) \Rightarrow V_a = V_t \ln\left(\frac{I_D}{I_S}\right)$$

so

$$V_a = (0.0259)\ln\left(\frac{0.539x10^{-3}}{2x10^{-11}}\right) \Rightarrow$$

$$\underline{V_a = 0.443\ V}$$

8.30

(a) $$\frac{1}{r_d} = \frac{dI_D}{dV_a} = I_S\left(\frac{1}{V_t}\right)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{0.020}{0.0259}\right)$$

which yields

$$\underline{r_d = 1.2x10^{11}\ \Omega}$$

(b)
For $V_a = -0.020\ V$,

$$\frac{1}{r_d} = \frac{10^{-13}}{0.0259}\cdot\exp\left(\frac{-0.020}{0.0259}\right)$$

or

$$\underline{r_d = 5.6x10^{11}\ \Omega}$$

8.31

Ideal reverse-saturation current density

$$J_S = \frac{eD_n n_{pO}}{L_n} + \frac{eD_p p_{nO}}{L_p}$$

We find

$$n_{pO} = \frac{n_i^2}{N_a} = \frac{(1.8x10^6)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

and

$$p_{nO} = \frac{(1.8x10^6)^2}{10^{16}} = 3.24x10^{-4}\ cm^{-3}$$

Also

$$L_n = \sqrt{D_n\tau_{nO}} = \sqrt{(200)(10^{-8})} = 14.2\ \mu m$$

$$L_p = \sqrt{D_p\tau_{pO}} = \sqrt{(6)(10^{-8})} = 2.45\ \mu m$$

Then

$$J_S = \frac{(1.6x10^{-19})(200)(3.24x10^{-4})}{14.2x10^{-4}} + \frac{(1.6x10^{-19})(6)(3.24x10^{-4})}{2.45x10^{-4}}$$

so

$$\underline{J_S = 8.57x10^{-18}\ A/cm^2}$$

Reverse-biased generation current density

$$J_{gen} = \frac{en_i W}{2\tau_O}$$

We have

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(10^{16})(10^{16})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.16\ V$$

And

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.16+5)}{1.6x10^{-19}} \times\left[\frac{10^{16}+10^{16}}{(10^{16})(10^{16})}\right]\right]^{1/2}$$

or

$$W = 1.34x10^{-4}\ cm$$

Then

$$J_{gen} = \frac{(1.6x10^{-19})(1.8x10^6)(1.34x10^{-4})}{2(10^{-8})}$$

or

$$\underline{J_{gen} = 1.93x10^{-9}\ A/cm^2}$$

Generation current dominates in GaAs reverse-biased junctions.

8.32

(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= n_i^2\left(1.6x10^{19}\right)\left[\frac{1}{10^{16}}\sqrt{\frac{25}{5x10^{-7}}} + \frac{1}{10^{16}}\sqrt{\frac{10}{5x10^{-7}}}\right]$$

or

$$J_S = n_i^2\left(1.85x10^{-31}\right)$$

We also have

$$J_{gen} = \frac{en_iW}{2\tau_O}$$

For $V_{bi} + V_R = 5\,V$, we find $W = 1.14x10^{-4}\ cm$

So

$$J_{gen} = \frac{\left(1.6x10^{-19}\right)\left(1.14x10^{-4}\right)n_i}{2\left(5x10^{-7}\right)}$$

or

$$J_{gen} = n_i\left(1.82x10^{-17}\right)$$

When $J_S = J_{gen}$,

$$1.85x10^{-31}n_i = 1.82x10^{-17}$$

which yields

$$n_i = 9.88x10^{13}\ cm^{-3}$$

We have

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\left(9.88x10^{13}\right)^2 = \left(2.8x10^{19}\right)\left(1.04x10^{19}\right)\left(\frac{T}{300}\right)^3 \times \exp\left[\frac{-1.12}{(0.0259)(T/300)}\right]$$

By trial and error, we find

$$\underline{T = 505K}$$

At this temperature

$$J_S = J_{gen} = \left(1.82x10^{-17}\right)\left(9.88x10^{13}\right) \Rightarrow$$

$$\underline{J_S = J_{gen} = 1.8x10^{-3}\ A/cm^2}$$

(b)

$$J_S \exp\left(\frac{V_a}{V_t}\right) = J_{gen}\exp\left(\frac{V_a}{2V_t}\right)$$

At $T = 300K$

$$J_S = \left(1.5x10^{10}\right)^2\left(1.85x10^{-31}\right)$$

or

$$J_S = 4.16x10^{-11}\ A/cm^2$$

and

$$J_{gen} = \left(1.5x10^{10}\right)\left(1.82x10^{-17}\right) \Rightarrow$$

or

$$J_{gen} = 2.73x10^{-7}\ A/cm^2$$

Then we can write

$$\exp\left(\frac{V_a}{2V_t}\right) = \frac{J_{gen}}{J_S} = \frac{2.73x10^{-7}}{4.16x10^{-11}} = 6.56x10^3$$

so that

$$V_a = 2(0.0259)\ln\left(6.56x10^3\right) \Rightarrow$$

$$\underline{V_a = 0.455\,V}$$

8.33

(a) We can write

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

We find

$$D_n = (3000)(0.0259) = 77.7\ cm^2/s$$

$$D_p = (200)(0.0259) = 5.18\ cm^2/s$$

Then

$$J_S = \left(1.6x10^{-19}\right)\left(1.8x10^6\right)^2\left[\frac{1}{10^{17}}\sqrt{\frac{77.7}{10^{-8}}} + \frac{1}{10^{17}}\sqrt{\frac{5.18}{10^{-8}}}\right]$$

or

$$J_S = 5.75x10^{-19}\ A/cm^2$$

so

$$I_S = AJ_S = \left(10^{-3}\right)\left(5.75x10^{-19}\right)$$

or

$$\underline{I_S = 5.75x10^{-22}\ A}$$

We also have

$$I_{gen} = \frac{en_iWA}{2\tau_O}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(10^{17})(10^{17})}{(1.8x10^6)^2}\right]$$

or

$$V_{bi} = 1.28\,V$$

Also

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(1.28+5)}{1.6x10^{-19}} \times \left(\frac{10^{17}+10^{17}}{(10^{17})(10^{17})}\right)\right]^{1/2}$$

or

$$W = 0.427x10^{-4}\ cm$$

so

$$I_{gen} = \frac{(1.6x10^{-19})(1.8x10^6)(0.427x10^{-4})(10^{-3})}{2(10^{-8})}$$

or

$$\underline{I_{gen} = 6.15x10^{-13}\ A}$$

The total reverse-bias current

$$I_R = I_S + I_{gen} = 5.75x10^{-22} + 6.15x10^{-13}$$

or

$$\underline{I_R \approx 6.15x10^{-13}\ A}$$

Forward Bias: Ideal diffusion current

For $V_a = 0.3\,V$

$$I_D = I_S \exp\left(\frac{V_a}{V_t}\right) = (5.75x10^{-22})\exp\left(\frac{0.3}{0.0259}\right)$$

or

$$\underline{I_D = 6.17x10^{-17}\ A}$$

For $V_a = 0.5\,V$

$$I_D = (5.75x10^{-22})\exp\left(\frac{0.5}{0.0259}\right)$$

or

$$\underline{I_D = 1.39x10^{-13}\ A}$$

Recombination current

For $V_a = 0.3\,V$:

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.3)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.169x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{en_i WA}{2\tau_O}\exp\left(\frac{V_a}{2V_t}\right)$$

$$= \frac{(1.6x10^{-19})(1.8x10^6)(0.169x10^{-4})(10^{-3})}{2(10^{-8})} \times \exp\left[\frac{0.3}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 7.96x10^{-11}\ A}$$

For $V_a = 0.5\,V$

$$W = \left[\frac{2(13.1)(8.85x10^{-14})(1.28-0.5)}{1.6x10^{-19}}\left(\frac{2x10^{17}}{10^{34}}\right)\right]^{1/2}$$

or

$$W = 0.150x10^{-4}\ cm$$

Then

$$I_{rec} = \frac{(1.6x10^{-19})(1.8x10^6)(0.15x10^{-4})(10^{-3})}{2(10^{-8})} \times \exp\left[\frac{0.5}{2(0.0259)}\right]$$

or

$$\underline{I_{rec} = 3.36x10^{-9}\ A}$$

Total forward-bias current:

For $V_a = 0.3\,V$;

$$I_D = 6.17x10^{-17} + 7.96x10^{-11}$$

or

$$\underline{I_D \approx 7.96x10^{-11}\ A}$$

For $V_a = 0.5\,V$

$$I_D = 1.39x10^{-13} + 3.36x10^{-9}$$

or

$$\underline{I_D \approx 3.36x10^{-9}\ A}$$

(b)

Reverse-bias; ratio of generation to ideal diffusion current:

$$\frac{I_{gen}}{I_S} = \frac{6.15x10^{-13}}{5.75x10^{-22}}$$

Ratio $\underline{= 1.07x10^{9}}$

Forward bias: Ratio of recombination to ideal diffusion current:

For $V_a = 0.3\ V$

$$\frac{I_{rec}}{I_D} = \frac{7.96x10^{-11}}{6.17x10^{-17}}$$

Ratio $\underline{= 1.29x10^{6}}$

For $V_a = 0.5\ V$

$$\frac{I_{rec}}{I_D} = \frac{3.36x10^{-9}}{1.39x10^{-13}}$$

Ratio $\underline{= 2.42x10^{4}}$

8.34
Computer Plot

8.35
Computer Plot

8.36
Computer Plot

8.37
We have that

$$R = \frac{np - n_i^2}{\tau_{pO}(n+n') + \tau_{nO}(p+p')}$$

Let $\tau_{pO} = \tau_{nO} = \tau_O$ and $n' = p' = n_i$

We can write

$$n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

We also have

$$\left(E_{Fn} - E_{Fi}\right) + \left(E_{Fi} - E_{Fp}\right) = eV_a$$

so that

$$\left(E_{Fi} - E_{Fp}\right) = eV_a - \left(E_{Fn} - E_{Fi}\right)$$

Then

$$p = n_i \exp\left[\frac{eV_a - \left(E_{Fn} - E_{Fi}\right)}{kT}\right]$$

$$= n_i \exp\left(\frac{eV_a}{kT}\right)\cdot \exp\left[\frac{-\left(E_{Fn} - E_{Fi}\right)}{kT}\right]$$

Define

$$\eta_a = \frac{eV_a}{kT} \quad \text{and} \quad \eta = \left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Then the recombination rate can be written as

$$R = \frac{\left(n_i e^{\eta}\right)\left(n_i e^{\eta_a}\cdot e^{-\eta}\right) - n_i^2}{\tau_O\left[n_i e^{\eta} + n_i + n_i e^{\eta_a}\cdot e^{-\eta} + n_i\right]}$$

or

$$R = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left(2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right)}$$

To find the maximum recombination rate, set

$$\frac{dR}{d\eta} = 0$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot\frac{d}{dx}\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^{-1}$$

or

$$0 = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot(-1)\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^{-2}$$

$$\times\left[e^{\eta} - e^{\eta_a}\cdot e^{-\eta}\right]$$

which simplifies to

$$0 = \frac{-n_i\left(e^{\eta_a} - 1\right)}{\tau_O}\cdot\frac{\left[e^{\eta} - e^{\eta_a}\cdot e^{-\eta}\right]}{\left[2 + e^{\eta} + e^{\eta_a}\cdot e^{-\eta}\right]^2}$$

The denominator is not zero, so we have

$$e^{\eta} - e^{\eta_a}\cdot e^{-\eta} = 0 \Rightarrow$$

$$e^{2\eta} = e^{\eta_a} \Rightarrow \eta = \frac{1}{2}\eta_a$$

Then the maximum recombination rate becomes

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a}\cdot e^{-\eta_a/2}\right]}$$

$$= \frac{n_i\left(e^{\eta_a} - 1\right)}{\tau_O\left[2 + e^{\eta_a/2} + e^{\eta_a/2}\right]}$$

or

$$R_{\max} = \frac{n_i\left(e^{\eta_a} - 1\right)}{2\tau_O\left(e^{\eta_a/2} + 1\right)}$$

which can be written as

$$R_{\max} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau_O\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

If $V_a >> \left(\frac{kT}{e}\right)$, then we can neglect the (-1) term in the numerator and the (+1) term in the denominator so we finally have

$$R_{max} = \frac{n_i}{2\tau_O}\exp\left(\frac{eV_a}{2kT}\right)$$

Q.E.D.

8.38
We have

$$J_{gen} = \int_0^W eGdx$$

In this case, $G = g' = 4x10^{19}\ cm^{-3}s^{-1}$, that is a constant through the space charge region. Then

$$J_{gen} = eg'W$$

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= (0.0259)\ln\left[\frac{(5x10^{15})(5x10^{15})}{(1.5x10^{10})^2}\right] = 0.659\ V$$

and

$$W = \left[\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.659+10)}{1.6x10^{-19}} \times\left(\frac{5x10^{15}+5x10^{15}}{(5x10^{15})(5x10^{15})}\right)\right]^{1/2}$$

or

$$W = 2.35x10^{-4}\ cm$$

Then

$$J_{gen} = (1.6x10^{-19})(4x10^{19})(2.35x10^{-4})$$

or

$$J_{gen} = 1.5x10^{-3}\ A/cm^2$$

8.39

$$J_S = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{nO}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}\right]$$

$$= (1.6x10^{-19})(1.5x10^{10})^2\left[\frac{1}{3x10^{16}}\sqrt{\frac{18}{10^{-7}}} + \frac{1}{10^{18}}\sqrt{\frac{6}{10^{-7}}}\right]$$

or

$$J_S = 1.64x10^{-11}\ A/cm^2$$

Now

$$J_D = J_S\exp\left(\frac{V_D}{V_t}\right)$$

Also

$$J = 0 = J_G - J_D$$

or

$$0 = 25x10^{-3} - 1.64x10^{-11}\exp\left(\frac{V_D}{V_t}\right)$$

which yields

$$\exp\left(\frac{V_D}{V_t}\right) = 1.52x10^9$$

or

$$V_D = V_t\ln(1.52x10^9)$$

so

$$V_D = 0.548\ V$$

8.40

$$V_B = \frac{\in E_{crit}^2}{2eN_B}$$

or

$$30 = \frac{(11.7)(8.85x10^{-14})(4x10^5)^2}{2(1.6x10^{-19})N_B}$$

which yields

$$N_B = N_d = 1.73x10^{16}\ cm^{-3}$$

8.41
For the breakdown voltage, we need $N_d = 3x10^{15}\ cm^{-3}$ and for this doping, we find $\mu_p = 430\ cm^2/V-s$. Then

$$D_p = (430)(0.0259) = 11.14\ cm^2/s$$

For the p^+n junction,

$$J_S = en_i^2\cdot\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{pO}}}$$

$$= \frac{(1.6x10^{-19})(1.5x10^{10})^2}{3x10^{15}}\sqrt{\frac{11.14}{10^{-7}}}$$

or

$$J_S = 1.27x10^{-10}\ A/cm^2$$

Then

$$I = J_S A \exp\left(\frac{V_a}{V_t}\right)$$

$$2x10^{-3} = \left(1.27x10^{-10}\right) A \exp\left(\frac{0.65}{0.0259}\right)$$

Finally

$$\underline{A = 1.99x10^{-4}\ cm^2}$$

8.42

GaAs, n^+p, and $N_a = 10^{16}\ cm^{-3}$

From Figure 8.25

$$\underline{V_B \approx 75\ V}$$

8.43

$$\mathrm{E}_{max} = \frac{eN_d x_n}{\in}$$

We can write

$$x_n = \frac{\mathrm{E}_{max} \in}{eN_d}$$

$$= \frac{\left(4x10^5\right)(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(5x10^{16}\right)}$$

or

$$x_n = 5.18x10^{-5}\ cm$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{16}\right)\left(5x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.778\ V$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

or

$$\left(5.18x10^{-5}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(V_{bi}+V_R\right)\left(\frac{5x10^{16}}{5x10^{16}}\right)\left(\frac{1}{5x10^{16}+5x10^{16}}\right)\right]$$

which yields

$$2.68x10^{-9} = 1.29x10^{-10}\left(V_{bi}+V_R\right)$$

so

$$V_{bi}+V_R = 20.7 \Rightarrow \underline{V_R = 19.9\ V}$$

8.44

For a silicon p^+n junction with

$N_d = 5x10^{15}\ cm^{-3}$ and $V_B \approx 100\ V$

Neglecting V_{bi} compared to V_B

$$x_n \approx \left[\frac{2\in V_B}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(100)}{\left(1.6x10^{-19}\right)\left(5x10^{15}\right)}\right]^{1/2}$$

or

$$\underline{x_n(\min) = 5.09\ \mu m}$$

8.45

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{18}\right)\left(10^{18}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933\ V$$

Now

$$\mathrm{E}_{max} = \frac{eN_d x_n}{\in}$$

so

$$10^6 = \frac{\left(1.6x10^{-19}\right)\left(10^{18}\right)x_n}{(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$x_n = 6.47x10^{-6}\ cm$$

Now

$$x_n = \left[\frac{2\in\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

Then

$$\left(6.47x10^{-6}\right)^2 = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{1.6x10^{-19}}\right.$$

$$\left.\times\left(V_{bi}+V_R\right)\left(\frac{10^{18}}{10^{18}}\right)\left(\frac{1}{10^{18}+10^{18}}\right)\right]$$

which yields

$$V_{bi}+V_R = 6.468\ V$$

or

$$\underline{V_R = 5.54\ V}$$

8.46

Assume silicon: For an n^+p junction

$$x_p = \left[\frac{2\in\left(V_{bi}+V_R\right)}{eN_a}\right]^{1/2}$$

Assume $V_{bi} << V_R$
(a)
For $x_p = 75\ \mu m$
Then

$$\left(75x10^{-4}\right)^2 = \frac{2(11.7)\left(8.85x10^{-14}\right)V_R}{\left(1.6x10^{-19}\right)\left(10^{15}\right)}$$

which yields $V_R = 4.35x10^3\ V$
(b)
For $x_p = 150\ \mu m$, we find

$V_R = 1.74x10^4\ V$

From Figure 8.25, the breakdown voltage is approximately 300 V. So, in each case, breakdown is reached first.

8.47
Impurity gradient

$$a = \frac{2x10^{18}}{2x10^{-4}} = 10^{22}\ cm^{-4}$$

From the figure

$V_B = 15\ V$

8.48
(a) If $\frac{I_R}{I_F} = 0.2$
Then we have

$$erf\sqrt{\frac{t_s}{\tau_{pO}}} = \frac{I_F}{I_F + I_R} = \frac{1}{1+\frac{I_R}{I_F}} = \frac{1}{1+0.2}$$

or

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = 0.833$$

We find

$$\sqrt{\frac{t_S}{\tau_{pO}}} = 0.978 \Rightarrow \frac{t_S}{\tau_{pO}} = 0.956$$

(b)
If $\frac{I_R}{I_F} = 1.0$, then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1+1} = 0.5$$

which yields

$$\frac{t_S}{\tau_{pO}} = 0.228$$

8.49
We want

$$\frac{t_S}{\tau_{pO}} = 0.2$$

Then

$$erf\sqrt{\frac{t_S}{\tau_{pO}}} = \frac{1}{1+\frac{I_R}{I_F}} = erf\sqrt{0.2}$$

where

$$erf\sqrt{0.2} = erf(0.447) = 0.473$$

We obtain

$$\frac{I_R}{I_F} = \frac{1}{0.473} - 1 \Rightarrow \frac{I_R}{I_F} = 1.11$$

We have

$$erf\sqrt{\frac{t_2}{\tau_{pO}}} + \frac{\exp\left(\frac{-t_2}{\tau_{pO}}\right)}{\sqrt{\pi\left(\frac{t_2}{\tau_{pO}}\right)}} = 1 + (0.1)\left(\frac{I_R}{I_F}\right) = 1.11$$

By trial and error,

$$\frac{t_2}{\tau_{pO}} = 0.65$$

8.50
$C_j = 18\ pF$ at $V_R = 0$
$C_j = 4.2\ pF$ at $V_R = 10\ V$
We have $\tau_{nO} = \tau_{pO} = 10^{-7}\ s$, $I_F = 2\ mA$
And $I_R \approx \frac{V_R}{R} = \frac{10}{10} = 1\ mA$
So

$$t_S \approx \tau_{pO} \ln\left(1 + \frac{I_F}{I_R}\right) = \left(10^{-7}\right)\ln\left(1 + \frac{2}{1}\right)$$

or

$$t_S = 1.1x10^{-7}\ s$$

Also

$$C_{avg} = \frac{18 + 4.2}{2} = 11.1\ pF$$

The time constant is

$$\tau_S = RC_{avg} = (10^4)(11.1x10^{-12}) = 1.11x10^{-7} \ s$$

Now

Turn-off time $= t_S + \tau_S = (1.1 + 1.11) \times 10^{-7} \ s$

Or

$\underline{2.21x10^{-7} \ s}$

8.51

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{19})^2}{(1.5x10^{10})^2}\right] = 1.14 \ V$$

We find

$$W = \left[\frac{2\in(V_{bi} - V_a)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(1.14 - 0.40)}{1.6x10^{-19}} \times \left(\frac{5x10^{19} + 5x10^{19}}{(5x10^{19})^2}\right)\right]^{1/2}$$

which yields

$\underline{W = 6.19x10^{-7} \ cm = 61.9 \ \mathring{A}}$

8.52

Sketch

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