Chapter 9

Problem Solutions

9.1

(a) We have

$$e\phi_n = eV_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ eV$$

(c)

$$\phi_{BO} = \phi_m - \chi = 4.28 - 4.01$$

or

$$\underline{\phi_{BO} = 0.27\ V}$$

and

$$V_{bi} = \phi_{BO} - \phi_n = 0.27 - 0.206$$

or

$$V_{bi} = 0.064\ V$$

Also

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.064)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 9.1x10^{-6}\ cm}$$

Then

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(9.1x10^{-6})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.41x10^4\ V / cm}$$

(d)

Using the figure, $\underline{\phi_{Bn} = 0.55\ V}$

So

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$V_{bi} = 0.344\ V$$

We then find

$\underline{x_d = 2.11x10^{-5}\ cm}$ and $\underline{E_{max} = 3.26x10^4\ V / cm}$

9.2

(a) $\phi_{BO} = \phi_m - \chi = 5.1 - 4.01$

or

$$\underline{\phi_{BO} = 1.09\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{15}}\right) = 0.265\ V$$

Then

$$V_{bi} = \phi_{BO} - \phi_n = 1.09 - 0.265$$

or

$$\underline{V_{bi} = 0.825\ V}$$

(c)

$$W = x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.825)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$\underline{W = 1.03x10^{-4}\ cm}$$

(d)

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{15})(1.03x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.59x10^4\ V / cm}$$

9.3

(a) Gold on n-type GaAs

$\chi = 4.07\ V$ and $\phi_m = 5.1\ V$

$$\phi_{BO} = \phi_m - \chi = 5.1 - 4.07$$

and

$$\underline{\phi_{BO} = 1.03\ V}$$

(b)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0580\ V}$$

(c)

$$V_{bi} = \phi_{BO} - \phi_n = 1.03 - 0.058$$

or

$$\underline{V_{bi} = 0.972\ V}$$

(d)

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.972+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.416\ \mu m}$$

(e)

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.416x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.87x10^5\ V/cm}$$

9.4

$\phi_{Bn} = 0.86\ V$ and $\phi_n = 0.058\ V$ (Problem 9.3)

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.86 - 0.058$$

or

$$\underline{V_{bi} = 0.802\ V}$$

and

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.802+5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or]

$$\underline{x_d = 0.410\ \mu m}$$

Also

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.410x10^{-4})}{(13.1)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 2.83x10^5\ V/cm}$$

9.5

Gold, n-type silicon junction. From the figure,

$$\phi_{Bn} = 0.81\ V$$

For $N_d = 5x10^{15}\ cm^{-3}$, we have

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{15}}\right) = \phi_n = 0.224\ V$$

Then

$$V_{bi} = 0.81 - 0.224 = 0.586\ V$$

(a)

Now

$$C' = \left[\frac{e\in N_d}{2(V_{bi}+V_R)}\right]^{1/2}$$

$$= \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{15})}{2(0.586+4)}\right]^{1/2}$$

or

$$C' = 9.50x10^{-9}\ F/cm^2$$

For $A = 5x10^{-4}\ cm^2$, $C = C'A$

So

$$\underline{C = 4.75\ pF}$$

(b)

For $N_d = 5x10^{16}\ cm^{-3}$, we find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{5x10^{16}}\right) = 0.164\ V$$

Then

$$V_{bi} = 0.81 - 0.164 = 0.646\ V$$

Now

$$C' = \left[\frac{(1.6x10^{-19})(11.7)(8.85x10^{-14})(5x10^{16})}{2(0.646+4)}\right]^{1/2}$$

or

$$C' = 2.99x10^{-8}\ F/cm^2$$

and

$$C = C'A$$

so

$$\underline{C = 15\ pF}$$

9.6

(a) From the figure, $\underline{V_{bi} = 0.90\ V}$

(b) We find

$$\frac{\Delta\left(\frac{1}{C'}\right)^2}{\Delta V_R} = \frac{3x10^{15} - 0}{2 - (-0.9)} = 1.03x10^{15}$$

and

$$1.03x10^{15} = \frac{2}{e \in N_d}$$

Then we can write

$$N_d = \frac{2}{(1.6x10^{-19})(13.1)(8.85x10^{-14})(1.03x10^{15})}$$

or

$$\underline{N_d = 1.05x10^{16}\ cm^{-3}}$$

(c)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{1.05x10^{16}}\right)$$

or

$$\underline{\phi_n = 0.0985\ V}$$

(d)

$$\phi_{Bn} = V_{bi} + \phi_n = 0.90 + 0.0985$$

or

$$\underline{\phi_{Bn} = 0.9985\ V}$$

9.7

From the figure, $\phi_{Bn} = 0.55\ V$

(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.55 - 0.206$$

or

$$\underline{V_{bi} = 0.344\ V}$$

We find

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.344)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.211\ \mu m}$$

Also

$$|E_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.211x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 3.26x10^4\ V/cm}$$

(b)

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \in}} = \left[\frac{(1.6x10^{-19})(3.26x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\underline{\Delta\phi = 20.0\ mV}$$

Also

$$x_m = \sqrt{\frac{e}{16\pi \in E}}$$

$$= \left[\frac{(1.6x10^{-19})}{16\pi(11.7)(8.85x10^{-14})(3.26x10^4)}\right]^{1/2}$$

or

$$\underline{x_m = 0.307x10^{-6}\ cm}$$

(c)

For $V_R = 4\ V$

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(0.344 + 4)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.75\ \mu m}$$

and

$$|E_{max}| = \frac{(1.6x10^{-19})(10^{16})(0.75x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$\underline{|E_{max}| = 1.16x10^5\ V/cm}$$

We find

$$\Delta\phi = \sqrt{\frac{eE}{4\pi \in}} \Rightarrow \underline{\Delta\phi = 37.8\ mV}$$

and

$$x_m = \sqrt{\frac{e}{16\pi \in \text{E}}} \Rightarrow \underline{x_m = 0.163x10^{-6}\ cm}$$

9.8
We have

$$-\phi(x) = \frac{-e}{16\pi \in x} - \text{E}x$$

or

$$e\phi(x) = \frac{e^2}{16\pi \in x} + \text{E}ex$$

Now

$$\frac{d(e\phi(x))}{dx} = 0 = \frac{-e^2}{16\pi \in x^2} + \text{E}e$$

Solving for x^2, we find

$$x^2 = \frac{e}{16\pi \in \text{E}}$$

or

$$\underline{x = x_m = \sqrt{\frac{e}{16\pi \in \text{E}}}}$$

Substituting this value of $x_m = x$ into the equation for the potential, we find

$$\Delta\phi = \frac{e}{16\pi \in \sqrt{\frac{e}{16\pi \in \text{E}}}} + \text{E}\sqrt{\frac{e}{16\pi \in \text{E}}}$$

which yields

$$\underline{\Delta\phi = \sqrt{\frac{e\text{E}}{4\pi \in}}}$$

9.9
Gold, n-type GaAs, from the figure $\phi_{Bn} = 0.87\ V$
(a)

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = \phi_{Bn} - \phi_n = 0.87 - 0.058$$

or

$$\underline{V_{bi} = 0.812\ V}$$

Also

$$x_d = \left[\frac{2 \in V_{bi}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.812)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$\underline{x_d = 0.153\ \mu m}$$

Then

$$|\text{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \left[\frac{(1.6x10^{-19})(5x10^{16})(0.153x10^{-4})}{(13.1)(8.85x10^{-14})}\right]$$

or

$$\underline{|\text{E}_{max}| = 1.06x10^5\ V/cm}$$

(b)
We want $\Delta\phi$ to be 7% or ϕ_{Bn},
So

$$\Delta\phi = (0.07)(0.87) = 0.0609\ V$$

Now

$$\Delta\phi = \sqrt{\frac{e\text{E}}{4\pi \in}} \Rightarrow \text{E} = \frac{(\Delta\phi^2)(4\pi \in)}{e}$$

so

$$\text{E} = \frac{(0.0609)^2(4\pi)(13.1)(8.85x10^{-14})}{1.6x10^{-19}}$$

or

$$\text{E}_{max} = 3.38x10^5\ V/cm$$

Now

$$\text{E}_{max} = \frac{eN_d x_d}{\in} \Rightarrow x_d = \frac{\in \text{E}}{eN_d}$$

so

$$x_d = \frac{(13.1)(8.85x10^{-14})(3.38x10^5)}{(1.6x10^{-19})(5x10^{16})}$$

or

$$x_d = 0.49\ \mu m$$

Then

$$x_d = 0.49x10^{-4} = \left[\frac{2 \in (V_{bi} + V_R)}{eN_d}\right]^{1/2}$$

or we can write

$$(V_{bi} + V_R) = \frac{eN_d x_d^2}{2 \in}$$

$$= \frac{(1.6x10^{-19})(5x10^{16})(0.49x10^{-4})^2}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{bi} + V_R = 8.28\ V = 0.812 + V_R$$

or

$$\underline{V_R = 7.47\ V}$$

9.10

Computer Plot

9.11

(a) $\phi_{BO} = \phi_m - \chi = 5.2 - 4.07$

or

$$\underline{\phi_{BO} = 1.13\ V}$$

(b)

We have

$$\left(E_g - e\phi_O - e\phi_{Bn}\right) = \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$
$$- \frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

which becomes

$$e\left(1.43 - 0.60 - \phi_{Bn}\right)$$
$$= \frac{1}{e\left(\frac{10^{13}}{e}\right)}\left[2\left(1.6x10^{-19}\right)(13.1)\left(8.85x10^{-14}\right)\right.$$
$$\left.\times\left(10^{16}\right)\left(\phi_{Bn} - 0.10\right)\right]^{1/2}$$
$$- \frac{\left(8.85x10^{-14}\right)}{e\left(\frac{10^{13}}{e}\right)\left(25x10^{-8}\right)}\left[5.2 - \left(4.07 + \phi_{Bn}\right)\right]$$

or

$$0.83 - \phi_{Bn}$$
$$= 0.038\sqrt{\phi_{Bn} = 0.10} - 0.221\left(1.13 - \phi_{Bn}\right)$$

We then find

$$\underline{\phi_{Bn} = 0.858\ V}$$

(c)

If $\phi_m = 4.5\ V$, then

$$\phi_{BO} = \phi_m - \chi = 4.5 - 4.07$$

or

$$\underline{\phi_{BO} = 0.43\ V}$$

From part (b), we have

$$0.83 - \phi_{Bn}$$
$$= 0.038\sqrt{\phi_{Bn} - 0.10} - 0.221\left[4.5 - \left(4.07 + \phi_{Bn}\right)\right]$$

We then find

$$\underline{\phi_{Bn} = 0.733\ V}$$

With interface states, the barrier height is less sensitive to the metal work function.

9.12

We have that

$$\left(E_g - e\phi_O - e\phi_{Bn}\right)$$
$$= \frac{1}{eD_{it}}\sqrt{2e \in N_d\left(\phi_{Bn} - \phi_n\right)}$$
$$- \frac{\in_i}{eD_{it}\delta}\left[\phi_m - \left(\chi + \phi_{Bn}\right)\right]$$

Let $eD_{it} = D'_{it}\left(cm^{-2}eV^{-1}\right)$. Then we can write

$$e(1.12 - 0.230 - 0.60)$$
$$= \frac{1}{D'_{it}}\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\right.$$
$$\left.\times\left(5x10^{16}\right)(0.60 - 0.164)\right]^{1/2}$$
$$- \frac{\left(8.85x10^{-14}\right)}{D'_{it}\left(20x10^{-8}\right)}\left[4.75 - (4.01 + 0.60)\right]$$

We find that

$$\underline{D'_{it} = 4.97x10^{11}\ cm^{-2}eV^{-1}}$$

9.13

(a) $\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right)$$

or

$$\underline{\phi_n = 0.206\ V}$$

(b)

$$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.206$$

or

$$\underline{V_{bi} = 0.684\ V}$$

(c)

$$J_{ST} = A^*T^2\exp\left(\frac{-e\phi_{Bn}}{kT}\right)$$

For silicon, $A^* = 120\ A/cm^2/°K^2$

Then

$$J_{ST} = (120)(300)^2\exp\left(\frac{-0.89}{0.0259}\right)$$

or

$$\underline{J_{ST} = 1.3x10^{-8}\ A/cm^2}$$

(d)

$$J_n = J_{ST} \exp\left(\frac{eV_a}{kT}\right)$$

or

$$V_a = V_t \ln\left(\frac{J_n}{J_{ST}}\right) = (0.0259)\ln\left(\frac{2}{1.3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.488\ V}$$

9.14

(a) From the figure, $\phi_{Bn} = 0.68\ V$

Then

$$J_{ST} = A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (120)(300)^2 \exp\left(\frac{-0.68}{0.0259}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\ A/cm^2$$

For $I = 10^{-3}\ A \Rightarrow J_n = \dfrac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$

We have

$$V_a = V_t \ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.278\ V}$$

For $I = 10\ mA \Rightarrow J_n = 20\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{20}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.338\ V}$$

For $I = 100\ mA \Rightarrow J_n = 200\ A/cm^2$

And

$$V_a = (0.0259)\ln\left(\frac{200}{4.28x10^{-5}}\right)$$

or

$$\underline{V_a = 0.398\ V}$$

(b)

For $T = 400K$, $\phi_{Bn} = 0.68\ V$

Now

$$J_{ST} = (120)(400)^2 \exp\left[\frac{-0.68}{(0.0259)(400/300)}\right]$$

or

$$J_{ST} = 5.39x10^{-2}\ A/cm^2$$

For $I = 1\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{2}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.125\ V}$$

For $I = 10\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left[\frac{20}{5.39x10^{-2}}\right]$$

or

$$\underline{V_a = 0.204\ V}$$

For $I = 100\ mA$,

$$V_a = (0.0259)\left(\frac{400}{300}\right)\ln\left(\frac{200}{5.39x10^{-2}}\right)$$

or

$$\underline{V_a = 0.284\ V}$$

9.15

(a) From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right)$$

$$= (1.12)(300)^2 \exp\left(\frac{-0.86}{0.0259}\right)$$

or

$$J_{ST} = 3.83x10^{-10}\ A/cm^2$$

Now

$$J_n = J_{ST} \exp\left(\frac{V_a}{V_t}\right)$$

and we can write, for $J_n = 5\ A/cm^2$

$$V_a = V_t \ln\left(\frac{J_n}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{5}{3.83x10^{-10}}\right)$$

or

$$\underline{V_a = 0.603\ V}$$

(b)
For $J_n = 10\ A/cm^2$

$$V_a = (0.0259)\ln\left(\frac{10}{3.83x10^{-10}}\right) = 0.621\ V$$

so

$$\Delta V_a = 0.621 - 0.603 \Rightarrow$$

$$\underline{\Delta V_a = 18\ mV}$$

9.16
Computer Plot

9.17
From the figure, $\phi_{Bn} = 0.86\ V$

$$J_{ST} = A^* T^2 \exp\left(\frac{-\phi_{Bn}}{V_t}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

$$= (120)(300)^2 \exp\left(\frac{-0.68}{0.0259}\right)\exp\left(\frac{\Delta\phi}{V_t}\right)$$

or

$$J_{ST} = 4.28x10^{-5}\exp\left(\frac{\Delta\phi}{V_t}\right)$$

We have

$$\Delta\phi = \sqrt{\frac{e\mathrm{E}}{4\pi\in}}$$

Now

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$$V_{bi} = \phi_{Bn} - \phi_n = 0.68 - 0.206 = 0.474\ V$$

(a)
We find for $V_R = 2\ V$,

$$x_d = \left[\frac{2\in(V_{bi}+V_R)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(2.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.566\ \mu m$$

Then

$$|\mathrm{E}_{max}| = \frac{eN_d x_d}{\in}$$

$$= \frac{(1.6x10^{-19})(10^{16})(0.566x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{max}| = 8.75x10^4\ V/cm$$

Now

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(8.75x10^4)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0328\ V$$

Then

$$J_{R1} = 4.28x10^{-5}\exp\left(\frac{0.0328}{0.0259}\right)$$

or

$$J_{R1} = 1.52x10^{-4}\ A/cm^2$$

For $A = 10^{-4}\ cm^2$, then

$$\underline{I_{R1} = 1.52x10^{-8}\ A}$$

(b)
For $V_R = 4\ V$,

$$x_d = \left[\frac{2(11.7)(8.85x10^{-14})(4.474)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_d = 0.761\ \mu m$$

$$|\mathrm{E}_{max}| = \frac{(1.6x10^{-19})(10^{16})(0.761x10^{-4})}{(11.7)(8.85x10^{-14})}$$

or

$$|\mathrm{E}_{max}| = 1.18x10^5\ V/cm$$

and

$$\Delta\phi = \left[\frac{(1.6x10^{-19})(1.18x10^5)}{4\pi(11.7)(8.85x10^{-14})}\right]^{1/2}$$

or

$$\Delta\phi = 0.0381\ V$$

Now

$$J_{R2} = 4.28x10^{-5}\exp\left(\frac{0.0381}{0.0259}\right)$$

or

$$J_{R2} = 1.86x10^{-4}\ A/cm^2$$

Finally,

$$\underline{I_{R2} = 1.86x10^{-8}\ A}$$

9.18
We have that

$$J_{s\to m}^{-} = \int_{E_C}^{\infty} v_x\,dn$$

The incremental electron concentration is given by

$$dn = g_C(E) f_F(E) dE$$

We have

$$g_C(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}$$

and, assuming the Boltzmann approximation

$$f_F(E) = \exp\left[\frac{-(E - E_F)}{kT}\right]$$

Then

$$dn = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_C}\cdot\exp\left[\frac{-(E - E_F)}{kT}\right]dE$$

If the energy above E_C is kinetic energy, then

$$\frac{1}{2}m_n^* v^2 = E - E_C$$

We can then write

$$\sqrt{E - E_C} = v\sqrt{\frac{m_n^*}{2}}$$

and

$$dE = \frac{1}{2}m_n^*\cdot 2v\,dv = m_n^* v\,dv$$

We can also write

$$E - E_F = (E - E_C) + (E_C - E_F)$$
$$= \frac{1}{2}m_n^* v^2 + e\phi_n$$

so that

$$dn = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)\cdot\exp\left(\frac{-m_n^* v^2}{2kT}\right)\cdot 4\pi v^2 dv$$

We can write

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

The differential volume element is

$$4\pi v^2 dv = dv_x dv_y dv_z$$

The current is due to all x-directed velocities that are greater than v_{Ox} and for all y- and z-directed velocities. Then

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3 \exp\left(\frac{-e\phi_n}{kT}\right)$$
$$\times\int_{v_{Ox}}^{\infty} v_x \exp\left(\frac{-m_n^* v_x^2}{2kT}\right)dv_x$$
$$\times\int_{-\infty}^{+\infty}\exp\left(\frac{-m_n^* v_y^2}{2kT}\right)dv_y \times\int_{-\infty}^{+\infty}\exp\left(\frac{-m_n^* v_z^2}{2kT}\right)dv_z$$

We can write that

$$\frac{1}{2}m_n^* v_{Ox}^2 = e(V_{bi} - V_a)$$

Make a change of variables:

$$\frac{m_n^* v_x^2}{2kT} = \alpha^2 + \frac{e(V_{bi} - V_a)}{kT}$$

or

$$v_x^2 = \frac{2kT}{m_n^*}\left[\alpha^2 + \frac{e(V_{bi} - V_a)}{kT}\right]$$

Taking the differential, we find

$$v_x dv_x = \left(\frac{2kT}{m_n^*}\right)\alpha\,d\alpha$$

We may note that when $v_x = v_{Ox}$, $\alpha = 0$.
Other change of variables:

$$\frac{m_n^* v_y^2}{2kT} = \beta^2 \Rightarrow v_y = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\beta$$

$$\frac{m_n^* v_z^2}{2kT} = \gamma^2 \Rightarrow v_z = \left(\frac{2kT}{m_n^*}\right)^{1/2}\cdot\gamma$$

Substituting the new variables, we have

$$J_{s\to m}^{-} = 2\left(\frac{m_n^*}{h}\right)^3\cdot\left(\frac{2kT}{m_n^*}\right)^2\exp\left(\frac{-e\phi_n}{kT}\right)$$
$$\times\exp\left[\frac{-e(V_{bi} - V_a)}{kT}\right]\cdot\int_0^{\infty}\alpha\exp\left(-\alpha^2\right)d\alpha$$
$$\times\int_{-\infty}^{+\infty}\exp\left(-\beta^2\right)d\beta\cdot\int_{-\infty}^{+\infty}\exp\left(-\gamma^2\right)d\gamma$$

9.19
For the Schottky diode,

$$J_{ST} = 3x10^{-8}\ A/cm^2\ ,\ A = 5x10^{-4}\ cm^2$$

For $I = 1\ mA$,

$$J = \frac{10^{-3}}{5x10^{-4}} = 2\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{2}{3x10^{-8}}\right)$$

or

$$\underline{V_a = 0.467\ V} \quad \text{(Schottky diode)}$$

For the pn junction, $J_S = 3x10^{-12}\ A/cm^2$

Then

$$V_a = (0.0259)\ln\left(\frac{2}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.705\ V} \quad \text{(pn junction diode)}$$

9.20

For the pn junction diode,

$$J_S = 5x10^{-12}\ A/cm^2\ ,\ A = 8x10^{-4}\ cm^2$$

For $I = 1.2\ mA$,

$$J = \frac{1.2x10^{-3}}{8x10^{-4}} = 1.5\ A/cm^2$$

Then

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

$$= (0.0259)\ln\left(\frac{1.5}{5x10^{-12}}\right) = 0.684\ V$$

For the Schottky diode, the applied voltage will be less, so

$$V_a = 0.684 - 0.265 = 0.419\ V$$

We have

$$I = AJ_{ST}\exp\left(\frac{V_a}{V_t}\right)$$

so

$$1.2x10^{-3} = A(7x10^{-8})\exp\left(\frac{0.419}{0.0259}\right)$$

which yields

$$\underline{A = 1.62x10^{-3}\ cm^2}$$

9.21

(a) Diodes in parallel:

We can write

$$I_S = I_{ST}\exp\left(\frac{V_{as}}{V_t}\right) \quad \text{(Schottky diode)}$$

and

$$I_{PN} = I_S\exp\left(\frac{V_{apn}}{V_t}\right) \quad \text{(pn junction diode)}$$

We have $I_S + I_{PN} = 0.5x10^{-3}\ A$, $V_{as} = V_{apn}$

Then

$$0.5x10^{-3} = (I_{ST} + I_S)\exp\left(\frac{V_a}{V_t}\right)$$

or

$$V_a = V_t \ln\left(\frac{0.5x10^{-3}}{I_S + I_{ST}}\right)$$

$$= (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8} + 10^{-12}}\right) = 0.239\ V$$

Now

$$I_S = 5x10^{-8}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_S \approx 0.5x10^{-3}\ A} \quad \text{(Schottky diode)}$$

and

$$I_{PN} = 10^{-12}\exp\left(\frac{0.239}{0.0259}\right)$$

or

$$\underline{I_{PN} = 1.02x10^{-8}\ A} \quad \text{(pn junction diode)}$$

(b) Diodes in Series:

We obtain,

$$V_{as} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{5x10^{-8}}\right)$$

or

$$\underline{V_{as} = 0.239\ V} \quad \text{(Schottky diode)}$$

and

$$V_{apn} = (0.0259)\ln\left(\frac{0.5x10^{-3}}{10^{-12}}\right)$$

or

$$\underline{V_{apn} = 0.519\ V} \quad \text{(pn junction diode)}$$

9.22

(a) For $I = 0.8\ mA$, we find

$$J = \frac{0.8x10^{-3}}{7x10^{-4}} = 1.14\ A/cm^2$$

We have

$$V_a = V_t \ln\left(\frac{J}{J_S}\right)$$

For the pn junction diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{3x10^{-12}}\right)$$

or

$$\underline{V_a = 0.691\ V}$$

For the Schottky diode,

$$V_a = (0.0259)\ln\left(\frac{1.14}{4x10^{-8}}\right)$$

or

$$\underline{V_a = 0.445\ V}$$

(b)
For the pn junction diode,

$$J_S \propto n_i^2 \propto \left(\frac{T}{300}\right)^3 \exp\left(\frac{-E_g}{kT}\right)$$

Then

$$\frac{J_S(400)}{J_S(300)}$$

$$= \left(\frac{400}{300}\right)^3 \exp\left[\frac{-E_g}{(0.0259)(400/300)} + \frac{E_g}{0.0259}\right]$$

or

$$= 2.37\exp\left[\frac{1.12}{0.0259} - \frac{1.12}{0.03453}\right]$$

We find

$$\frac{J_S(400)}{J_S(300)} = 1.16x10^5$$

Now

$$I = (7x10^{-4})(1.16x10^5)(3x10^{-12})\exp\left(\frac{0.691}{0.03453}\right)$$

or

$$\underline{I = 120\ mA}$$

For the Schottky diode

$$J_{ST} \propto T^2 \exp\left(\frac{-e\phi_{BO}}{kT}\right)$$

Now

$$\frac{J_{ST}(400)}{J_{ST}(300)}$$

$$= \left(\frac{400}{300}\right)^2 \exp\left[\frac{-\phi_{BO}}{(0.0259)(400/300)} + \frac{\phi_{BO}}{0.0259}\right]$$

or

$$= 1.78\exp\left[\frac{0.82}{0.0259} - \frac{0.82}{0.03453}\right]$$

We obtain

$$\frac{J_{ST}(400)}{J_{ST}(300)} = 4.85x10^3$$

and so

$$I = (7x10^{-4})(4.85x10^3)(4x10^{-8})\exp\left(\frac{0.445}{0.03453}\right)$$

or

$$\underline{I = 53.7\ mA}$$

9.23
Computer Plot

9.24
We have

$$R_C = \frac{\left(\frac{kT}{e}\right)\cdot\exp\left(\frac{e\phi_{Bn}}{kT}\right)}{A^*T^2}$$

which can be rewritten as

$$\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right] = \frac{e\phi_{Bn}}{kT}$$

so

$$\phi_{Bn} = \left(\frac{kT}{e}\right)\cdot\ln\left[\frac{R_C A^* T^2}{(kT/e)}\right]$$

$$= (0.0259)\ln\left[\frac{(10^{-5})(120)(300)^2}{0.0259}\right]$$

or

$$\underline{\phi_{Bn} = 0.216\ V}$$

9.25
(b) We need $\phi_n = \phi_m - \chi_s = 4.2 - 4.0 = 0.20\ V$
And

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

or

$$0.20 = (0.0259)\ln\left(\frac{2.8x10^{19}}{N_d}\right)$$

which yields

$$\underline{N_d = 1.24x10^{16}\ cm^{-3}}$$

(c)
Barrier height $\underline{= 0.20\ V}$

9.26
We have that

$$\mathrm{E} = \frac{-eN_d}{\in}(x_n - x)$$

Then

$$\phi = -\int \mathrm{E}dx = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right) + C_2$$

Let $\phi = 0$ at $x = 0 \Rightarrow C_2 = 0$
So

$$\phi = \frac{eN_d}{\in}\left(x_n \cdot x - \frac{x^2}{2}\right)$$

At $x = x_n$, $\phi = V_{bi}$, so

$$\phi = V_{bi} = \frac{eN_d}{\in} \cdot \frac{x_n^2}{2}$$

or

$$x_n = \sqrt{\frac{2 \in V_{bi}}{eN_d}}$$

Also

$$V_{bi} = \phi_{BO} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right)$$

For

$$\phi = \frac{\phi_{BO}}{2} = \frac{0.70}{2} = 0.35\ V$$

we have

$$0.35 = \frac{(1.6x10^{-19})N_d}{(11.7)(8.85x10^{-14})}\left[x_n(50x10^{-8}) - \frac{(50x10^{-8})^2}{2}\right]$$

or

$$0.35 = 7.73x10^{-14}\ N_d\left(x_n - 25x10^{-8}\right)$$

We have

$$x_n = \left[\frac{2(11.7)(8.85x10^{-14})V_{bi}}{(1.6x10^{-19})N_d}\right]^{1/2}$$

and

$$V_{bi} = 0.70 - \phi_n$$

By trial and error,

$$\underline{N_d = 3.5x10^{18}\ cm^{-3}}$$

9.27

(b) $\phi_{BO} = \phi_p = V_t \ln\left(\frac{N_V}{N_a}\right)$

$$= (0.0259)\ln\left(\frac{1.04x10^{19}}{5x10^{16}}\right) \Rightarrow$$

$$\underline{\phi_{BO} = 0.138\ V}$$

9.28
Sketches

9.29
Sketches

9.30
Electron affinity rule

$$\Delta E_C = e(\chi_n - \chi_p)$$

For GaAs, $\chi = 4.07$; and for AlAs, $\chi = 3.5$,
If we assume a linear extrapolation between GaAs and AlAs, then for

$$Al_{0.3}Ga_{0.7}As \Rightarrow \chi = 3.90$$

Then

$$|E_C| = 4.07 - 3.90 \Rightarrow$$

$$\underline{|E_C| = 0.17\ eV}$$

9.31
Consider an n-P heterojunction in thermal equilibrium. Poisson's equation is

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\in} = -\frac{d\mathrm{E}}{dx}$$

In the n-region,

$$\frac{d\mathrm{E}_n}{dx} = \frac{\rho(x)}{\in_n} = \frac{eN_{dn}}{\in_n}$$

For uniform doping, we have

$$\mathrm{E}_n = \frac{eN_{dn}x}{\in_n} + C_1$$

The boundary condition is
$\mathrm{E}_n = 0$ at $x = -x_n$, so we obtain

$$C_1 = \frac{eN_{dn}x_n}{\in_n}$$

Then

$$\mathrm{E}_n = \frac{eN_{dn}}{\in_n}(x + x_n)$$

In the P-region,

$$\frac{dE_P}{dx} = -\frac{eN_{aP}}{\in_P}$$

which gives

$$E_P = -\frac{eN_{aP}x}{\in_P} + C_2$$

We have the boundary condition that
$E_P = 0$ at $x = x_P$ so that

$$C_2 = \frac{eN_{aP}x_P}{\in_P}$$

Then

$$E_P = \frac{eN_{aP}}{\in_P}\left(x_P - x\right)$$

Assuming zero surface charge density at $x = 0$, the electric flux density D is continuous, so
$\in_n E_n(0) = \in_P E_P(0)$
which yields

$$N_{dn}x_n = N_{aP}x_P$$

We can determine the electric potential as

$$\phi_n(x) = -\int E_n dx$$

$$= -\left[\frac{eN_{dn}x^2}{2\in_n} + \frac{eN_{dn}x_n x}{\in_n}\right] + C_3$$

Now

$$V_{bin} = \left|\phi_n(0) - \phi_n(-x_n)\right|$$

$$= C_3 - \left[C_3 - \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{dn}x_n^2}{\in_n}\right]$$

or

$$V_{bin} = \frac{eN_n x_n^2}{2\in_n}$$

Similarly on the P-side, we find

$$V_{biP} = \frac{eN_{aP}x_p^2}{2\in_P}$$

We have that

$$V_{bi} = V_{bin} + V_{biP} = \frac{eN_{dn}x_n^2}{2\in_n} + \frac{eN_{aP}x_P^2}{2\in_P}$$

We can write

$$x_P = x_n\left(\frac{N_{dn}}{N_{aP}}\right)$$

Substituting and collecting terms, we find

$$V_{bi} = \left[\frac{e\in_P N_{dn}N_{aP} + e\in_n N_{dn}^2}{2\in_n\in_P N_{aP}}\right]\cdot x_n^2$$

Solving for x_n, we have

$$x_n = \left[\frac{2\in_n\in_P N_{aP}V_{bi}}{eN_{dn}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

Similarly on the P-side, we have

$$x_P = \left[\frac{2\in_n\in_P N_{dn}V_{bi}}{eN_{aP}\left(\in_P N_{aP} + \in_n N_{dn}\right)}\right]^{1/2}$$

The total space charge width is then

$$W = x_n + x_P$$

Substituting and collecting terms, we obtain

$$W = \left[\frac{2\in_n\in_P V_{bi}\left(N_{aP} + N_{dn}\right)}{eN_{dn}N_{aP}\left(\in_n N_{dn} + \in_P N_{aP}\right)}\right]^{1/2}$$