Chapter 10

Problem Solutions

10.1
Sketch

10.2
Sketch

10.3

(a) $|I_S| = \dfrac{eD_n A_{BE} n_{BO}}{x_B}$

$= \dfrac{(1.6x10^{-19})(20)(10^{-4})(10^4)}{10^{-4}}$

or $\underline{I_S = 3.2x10^{-14}\ A}$

(b)

(i) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.5}{0.0259}\right) \Rightarrow$

$\underline{i_C = 7.75\ \mu A}$

(ii) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.6}{0.0259}\right) \Rightarrow$

$\underline{i_C = 0.368\ mA}$

(iii) $i_C = 3.2x10^{-14}\exp\left(\dfrac{0.7}{0.0259}\right) \Rightarrow$

$\underline{i_C = 17.5\ mA}$

10.4

(a) $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9920}{1-0.9920} \Rightarrow \underline{\beta = 124}$

(b) From 10.3b

(i) For $i_C = 7.75\ \mu A$; $i_B = \dfrac{i_C}{\beta} = \dfrac{7.75}{124} \Rightarrow$

$\underline{i_B = 0.0625\ \mu A}$,

$i_E = \left(\dfrac{1+\beta}{\beta}\right)\cdot i_C = \left(\dfrac{125}{124}\right)(7.75) \Rightarrow$

$\underline{i_E = 7.81\ \mu A}$

(ii) For $i_C = 0.368\ mA$, $\underline{i_B = 2.97\ \mu A}$,

$\underline{i_E = 0.371\ mA}$

(iii) For $i_C = 17.5\ mA$, $\underline{i_B = 0.141\ mA}$,

$\underline{i_E = 17.64\ mA}$

10.5

(a) $\beta = \dfrac{i_C}{i_B} = \dfrac{510}{6} \Rightarrow \underline{\beta = 85}$

$\alpha = \dfrac{\beta}{1+\beta} = \dfrac{85}{86} \Rightarrow \underline{\alpha = 0.9884}$

$i_E = i_C + i_B = 510 + 6 \Rightarrow \underline{i_E = 516\ \mu A}$

(b)

$\beta = \dfrac{2.65}{0.05} \Rightarrow \underline{\beta = 53}$

$\alpha = \dfrac{53}{54} \Rightarrow \underline{\alpha = 0.9815}$

$i_E = 2.65 + 0.05 \Rightarrow \underline{i_E = 2.70\ mA}$

10.6

(c) For $i_B = 0.05\ mA$,

$i_C = \beta i_B = (100)(0.05) \Rightarrow \underline{i_C = 5\ mA}$

We have

$v_{CE} = V_{CC} - i_C R = 10 - (5)(1)$

or

$\underline{v_{CE} = 5\ V}$

10.7

(b) $V_{CC} = I_C R + V_{CB} + V_{BE}$

so

$10 = I_C(2) + 0 + 0.6$

or

$\underline{I_C = 4.7\ mA}$

10.8

(a)

$$n_{pO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

At $x = 0$,

$$n_p(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

or we can write

$$V_{BE} = V_t \ln\left(\frac{n_p(0)}{n_{pO}}\right)$$

We want $n_p(0) = 10\% \times 10^{16} = 10^{15}\ cm^{-3}$,
So

$$V_{BE} = (0.0259)\ln\left(\frac{10^{15}}{2.25x10^4}\right)$$

or

$$\underline{V_{BE} = 0.635\ V}$$

(b)
At $x' = 0$,

$$p_n(0) = p_{nO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$p_n(0) = 2.25x10^3\exp\left(\frac{0.635}{0.0259}\right) \Rightarrow$$

$$\underline{p_n(0) = 10^{14}\ cm^{-3}}$$

(c)
From the B-C space charge region,

$$x_{p1} = \left[\frac{2\in(V_{bi}+V_{R1})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right]^{1/2}$$

We find

$$V_{bi1} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right] = 0.635\ V$$

Then

$$x_{p1} = \left[\frac{2(11.7)(8.85x10^{-14})(0.635+3)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p1} = 0.207\ \mu m$$

We find

$$V_{bi2} = (0.0259)\ln\left[\frac{(10^{17})(10^{16})}{(1.5x10^{10})^2}\right] = 0.754\ V$$

Then

$$x_{p2} = \left[\frac{2(11.7)(8.85x10^{-14})(0.754-0.635)}{1.6x10^{-19}} \times\left(\frac{10^{17}}{10^{16}}\right)\left(\frac{1}{10^{17}+10^{16}}\right)\right]^{1/2}$$

or

$$x_{p2} = 0.118\ \mu m$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.10 - 0.207 - 0.118$$

or

$$\underline{x_B = 0.775\ \mu m}$$

10.9

(a) $$p_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{5x10^{17}} \Rightarrow$$

$$\underline{p_{EO} = 4.5x10^2\ cm^{-3}}$$

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} \Rightarrow$$

$$\underline{n_{BO} = 2.25x10^4\ cm^{-3}}$$

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$

$$\underline{p_{CO} = 2.25x10^5\ cm^{-3}}$$

(b)

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (2.25x10^4)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{n_B(0) = 6.80x10^{14}\ cm^{-3}}$$

Also

$$p_E(0) = p_{EO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= (4.5x10^2)\exp\left(\frac{0.625}{0.0259}\right)$$

or

$$\underline{p_E(0) = 1.36x10^{13}\ cm^{-3}}$$

10.10

(a) $$n_{EO} = \frac{n_i^2}{N_E} = \frac{(1.5x10^{10})^2}{10^{18}} \Rightarrow$$

$$\underline{n_{EO} = 2.25x10^2\ cm^{-3}}$$

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}} \Rightarrow$$

$p_{BO} = 4.5x10^3 \ cm^{-3}$

$$n_{CO} = \frac{n_i^2}{N_C} = \frac{(1.5x10^{10})^2}{10^{15}} \Rightarrow$$

$n_{CO} = 2.25x10^5 \ cm^{-3}$

(b)

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (4.5x10^3)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$p_B(0) = 3.57x10^{14} \ cm^{-3}$

Also

$$n_E(0) = n_{EO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25x10^2)\exp\left(\frac{0.650}{0.0259}\right)$$

or

$n_E(0) = 1.78x10^{13} \ cm^{-3}$

10.11

We have

$$\frac{d(\delta n_B)}{dx} = \frac{n_{BO}}{\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\left(\frac{-1}{L_B}\right)\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}\cosh\left(\frac{x}{L_B}\right)\right\}$$

At $x = 0$,

$$\frac{d(\delta n_B)}{dx}\Big|(0) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right.$$

$$\left.\times\cosh\left(\frac{x_B}{L_B}\right) + 1\right\}$$

At $x = x_B$,

$$\frac{d(\delta n_B)}{dx}\Big|(x_B) = \frac{-n_{BO}}{L_B\sinh\left(\frac{x_B}{L_B}\right)}$$

$$\times\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)\right\}$$

Taking the ratio,

$$\frac{\frac{d(\delta n_B)}{dx}\Big|(x_B)}{\frac{d(\delta n_B)}{dx}\Big|(0)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \cosh\left(\frac{x_B}{L_B}\right)}{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cosh\left(\frac{x_B}{L_B}\right) + 1}$$

$$\approx \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)}$$

(a) For $\frac{x_B}{L_B} = 0.1 \Rightarrow Ratio = 0.9950$

(b) For $\frac{x_B}{L_B} = 1.0 \Rightarrow Ratio = 0.648$

(c) For $\frac{x_B}{L_B} = 10 \Rightarrow Ratio = 9.08x10^{-5}$

10.12

In the base of the transistor, we have

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution to the differential equation is of the form,

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we have

$$\delta n_B(0) = A + B = n_B(0) - n_{BO}$$

$$= n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

Also

$$\delta n_B(x_B) = A\exp\left(\frac{x_B}{L_B}\right) + B\exp\left(\frac{-x_B}{L_B}\right) = -n_{BO}$$

From the first boundary condition, we can write

$$A = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] - B$$

Substituting into the second boundary condition equation, we find

$$B\left[\exp\left(\frac{x_B}{L_B}\right)-\exp\left(\frac{-x_B}{L_B}\right)\right]$$

$$=n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}$$

which can be written as

$$B=\frac{n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

We then find

$$A=\frac{-n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-n_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

10.13

In the base of the pnp transistor, we have

$$D_B\frac{d^2(\delta p_B(x))}{dx^2}-\frac{\delta p_B(x)}{\tau_{BO}}=0$$

or

$$\frac{d^2(\delta p_B(x))}{dx^2}-\frac{\delta p_B(x)}{L_B^2}=0$$

where $L_B=\sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta p_B(x)=A\exp\left(\frac{x}{L_B}\right)+B\exp\left(\frac{-x}{L_B}\right)$$

From the boundary conditions, we can write

$$\delta p_B(0)=A+B=p_B(0)-p_{BO}$$

$$=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]$$

Also

$$\delta p_B(x_B)=A\exp\left(\frac{x_B}{L_B}\right)+B\exp\left(\frac{-x_B}{L_B}\right)=-p_{BO}$$

From the first boundary condition equation, we find

$$A=p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]-B$$

Substituting into the second boundary equation

$$B=\frac{p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{x_B}{L_B}\right)+p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

and then we obtain

$$A=\frac{-p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x_B}{L_B}\right)-p_{BO}}{2\sinh\left(\frac{x_B}{L_B}\right)}$$

Substituting the expressions for A and B into the general solution and collecting terms, we obtain

$$\delta p_B(x)=p_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B-x}{L_B}\right)-\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

10.14

For the idealized straight line approximation, the total minority carrier concentration is given by

$$n_B(x)=n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)$$

The excess concentration is

$$\delta n_B=n_B(x)-n_{BO}$$

so for the idealized case, we can write

$$\delta n_{BO}(x)=n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]\cdot\left(\frac{x_B-x}{x_B}\right)-1\right\}$$

At $x=\frac{1}{2}x_B$, we have

$$\delta n_{BO}\left(\frac{1}{2}x_B\right)=n_{BO}\left\{\frac{1}{2}\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right]-1\right\}$$

For the actual case, we have

$$\delta n_B\left(\frac{1}{2}x_B\right)=n_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\sinh\left(\frac{x_B}{2L_B}\right)-\sinh\left(\frac{x_B}{2L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

(a) For $\frac{x_B}{L_B} = 0.1$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.0500208$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 0.100167$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right] \cdot (0.50 - 0.49937) - 1.0 + 0.99875}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$= \frac{(0.00063)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.00125)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

If we assume that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$, then we find that the ratio is

$$\frac{0.00063}{0.50} = 0.00126 \Rightarrow \underline{0.126\%}$$

(b)

For $\frac{x_B}{L_B} = 1.0$, we have

$$\sinh\left(\frac{x_B}{2L_B}\right) = 0.5211$$

and

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then

$$\frac{\delta n_{BO}\left(\frac{1}{2}x_B\right) - \delta n_B\left(\frac{1}{2}x_B\right)}{\delta n_{BO}\left(\frac{1}{2}x_B\right)}$$

$$= \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right)\right](0.50 - 0.4434) - 1.0 + 0.8868}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

which becomes

$$\frac{(0.0566)\exp\left(\frac{V_{BE}}{V_t}\right) - (0.1132)}{\frac{1}{2}\exp\left(\frac{V_{BE}}{V_t}\right) - 1}$$

Assuming that $\exp\left(\frac{V_{BE}}{V_t}\right) >> 1$

Then the ratio is

$$= \frac{0.0566}{0.50} = 0.1132 \Rightarrow \underline{11.32\%}$$

10.15

The excess hole concentration at $x = 0$ is

$$\delta p_B(0) = p_{BO}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] = 8x10^{14}\ cm^{-3}$$

and the excess hole concentration at $x = x_B$ is

$$\delta p_B(x_B) = -p_{BO} = -2.25x10^4\ cm^{-3}$$

From the results of problem 10.13, we can write

$$\delta p(x) = p_{BO}$$

$$\times\left\{\frac{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] \cdot \sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}\right\}$$

or

$$\delta p_B(x) =$$

$$\frac{(8x10^{14})\sinh\left(\frac{x_B - x}{L_B}\right) - (2.25x10^4)\sinh\left(\frac{x}{L_B}\right)}{\sinh\left(\frac{x_B}{L_B}\right)}$$

Let $x_B = L_B = 10\ \mu m$, so that

$$\sinh\left(\frac{x_B}{L_B}\right) = 1.1752$$

Then, we can find $\delta p_B(x)$ for (a) the ideal linear approximation and for (b) the actual distribution as follow:

x	$(a)\ \delta p_B$	$(b)\ \delta p_B$
0	$8x10^{14}$	$8x10^{14}$
$0.25L_B$	$6x10^{14}$	$5.6x10^{14}$
$0.50L_B$	$4x10^{14}$	$3.55x10^{14}$
$0.75L_B$	$2x10^{14}$	$1.72x10^{14}$
$1.0L_B$	$-2.25x10^{4}$	$-2.25x10^{4}$

(c)
For the ideal case when $x_B << L_B$, then $J(0) = J(x_B)$, so that

$$\frac{J(x_B)}{J(0)} = 1$$

For the case when $x_B = L_B = 10\ \mu m$

$$J(0) = \frac{eD_B}{\sinh\left(\frac{x_B}{L_B}\right)} \frac{d}{dx}\left\{\left(8x10^{14}\right)\sinh\left(\frac{x_B - x}{L_B}\right) - \left(2.25x10^{4}\right)\sinh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

or

$$J(0) = \frac{eD_B}{\sinh(1)}\left\{\frac{-1}{L_B}\left(8x10^{14}\right)\cosh\left(\frac{x_B - x}{L_B}\right) - \frac{1}{L_B}\left(2.25x10^{4}\right)\cosh\left(\frac{x}{L_B}\right)\right\}\Big|_{x=0}$$

which becomes

$$= \frac{-eD_B}{L_B\sinh(1)}\cdot\left\{\left(8x10^{14}\right)\cosh(1) + \left(2.25x10^{4}\right)\cosh(0)\right\}$$

We find

$$J(0) = \frac{-\left(1.6x10^{-19}\right)(10)}{\left(10x10^{-4}\right)(1.175)} \times\left[\left(8x10^{14}\right)(1.543) + \left(2.25x10^{4}\right)(1)\right]$$

or

$$J(0) = -1.68\ A/cm^2$$

Now

$$J(x_B) = \frac{-eD_B}{L_B\sinh(1)}\left\{\left(8x10^{14}\right)\cosh(0) + \left(2.25x10^{4}\right)\cosh(1)\right\}$$

or

$$= \frac{-\left(1.6x10^{-19}\right)(10)}{\left(10x10^{-4}\right)(1.175)} \times\left[\left(8x10^{14}\right)(1) + \left(2.25x10^{4}\right)(1.543)\right]$$

We obtain

$$J(x_B) = -1.089\ A/cm^2$$

Then

$$\frac{J(x_B)}{J(0)} = \frac{-1.089}{-1.68} \Rightarrow \frac{J(x_B)}{J(0)} = 0.648$$

10.16
(a) npn transistor biased in saturation

$$D_B\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{\tau_{BO}} = 0$$

or

$$\frac{d^2(\delta n_B(x))}{dx^2} - \frac{\delta n_B(x)}{L_B^2} = 0$$

where $L_B = \sqrt{D_B\tau_{BO}}$

The general solution is of the form

$$\delta n_B(x) = A\exp\left(\frac{x}{L_B}\right) + B\exp\left(\frac{-x}{L_B}\right)$$

If $x_B << L_B$, then also $x << L_B$ so that

$$\delta n_B(x) \approx A\left(1 + \frac{x}{L_B}\right) + B\left(1 - \frac{x}{L_B}\right)$$

$$= (A + B) + (A - B)\left(\frac{x}{L_B}\right)$$

which can be written as

$$\delta n_B(x) = C + D\left(\frac{x}{L_B}\right)$$

The boundary conditions are

$$\delta n_B(0) = C = n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]$$

and

$$\delta n_B(x_B) = C + D\left(\frac{x_B}{L_B}\right) = n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]$$

Then the coefficient D can be written as

$$D = \left(\frac{L_B}{x_B}\right)\left\{ n_{BO}\left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right] - n_{BO}\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\right\}$$

The excess electron concentration is then given by

$$\delta n_B(x) = n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(1 - \frac{x}{L_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{x}{x_B}\right)\right\}$$

(b)
The electron diffusion current density is

$$J_n = eD_B \frac{d(\delta n_B(x))}{dx}$$

$$= eD_B n_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(\frac{-1}{x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{1}{x_B}\right)\right\}$$

or

$$J_n = -\frac{eD_B n_{BO}}{x_B}\left\{\exp\left(\frac{V_{BE}}{V_t}\right) - \exp\left(\frac{V_{BC}}{V_t}\right)\right\}$$

(c)
The total excess charge in the base region is

$$Q_{nB} = -e\int_0^{x_B} \delta n_B(x)dx$$

$$= -en_{BO}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]\cdot\left(x - \frac{x^2}{2x_B}\right) + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\cdot\left(\frac{x^2}{2x_B}\right)\right\}\Bigg|_0^{x_B}$$

which yields

$$Q_{nB} = \frac{-en_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right] + \left[\exp\left(\frac{V_{BC}}{V_t}\right) - 1\right]\right\}$$

10.17
(a) Extending the results of problem 10.16 to a pnp transistor, we can write

$$J_P = \frac{eD_B p_{BO}}{x_B}\left[\exp\left(\frac{V_{EB}}{V_t}\right) - \exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$165 = \frac{\left(1.6x10^{-19}\right)(10)\left(2.25x10^3\right)}{0.7x10^{-4}} \times\left[\exp\left(\frac{0.75}{0.0259}\right) - \exp\left(\frac{V_{CB}}{V_t}\right)\right]$$

or

$$3.208x10^{12} = 3.768x10^{12} - \exp\left(\frac{V_{CB}}{V_t}\right)$$

which yields

$$V_{CB} = (0.0259)\ln\left(0.56x10^{12}\right) \Rightarrow$$

$$V_{CB} = 0.70\ V$$

(b)

$$V_{EC}(sat) = V_{EB} - V_{CB} = 0.75 - 0.70 \Rightarrow$$

$$V_{EC}(sat) = 0.05\ V$$

(c)
Again, extending the results of problem 10.16 to a pnp transistor, we can write

$$Q_{pB} = \frac{ep_{BO}x_B}{2}\left\{\left[\exp\left(\frac{V_{EB}}{V_t}\right) - 1\right] + \left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1\right]\right\}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(2.25x10^3\right)\left(0.7x10^{-4}\right)}{2} \times\left[3.768x10^{12} + 0.56x10^{12}\right]$$

or

$$Q_{pB} = 5.45x10^{-8}\ C/cm^2$$

or

$$\frac{Q_{pB}}{e} = 3.41x10^{11}\ holes/cm^2$$

(d)
In the collector, we have

$$\delta n_p(x) = n_{PO}\left[\exp\left(\frac{V_{CB}}{V_t}\right) - 1\right]\cdot\exp\left(\frac{-x}{L_C}\right)$$

The total number of excess electrons in the collector is

$$N_{Coll} = \int_0^\infty \delta n_P(x)dx$$

$$= -n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]\cdot\exp\left(\frac{-x}{L_C}\right)\Big|_0^\infty$$

$$= n_{PO}L_C\left[\exp\left(\frac{V_{CB}}{V_t}\right)-1\right]$$

We have

$$n_{PO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{15}} = 4.5x10^4\ cm^{-3}$$

Then the total number of electrons is

$$N_{Coll} = \left(4.5x10^4\right)\left(35x10^{-4}\right)\left(0.56x10^{12}\right)$$

or

$$\underline{N_{Coll} = 8.82x10^{13}\ electrons/cm^2}$$

10.18

(b) $n_{BO} = \dfrac{n_i^2}{N_B} = \dfrac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$

and

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{7x10^{15}} = 3.21x10^4\ cm^{-3}$$

At $x = x_B$,

$$n_B(x_B) = n_{BO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(2.25x10^3\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$n_B(x_B) = 6.7x10^{12}\ cm^{-3}$$

At $x'' = 0$,

$$p_C(0) = p_{CO}\exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= \left(3.21x10^4\right)\exp\left(\frac{0.565}{0.0259}\right)$$

or

$$p_C(0) = 9.56x10^{13}\ cm^{-3}$$

(c)
From the B-C space-charge region,

$$V_{bi1} = (0.0259)\ln\left[\frac{\left(10^{17}\right)\left(7x10^{15}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.745\ V$$

Then

$$x_{p1} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.745-0.565)}{1.6x10^{-19}}\times\left(\frac{7x10^{15}}{10^{17}}\right)\left(\frac{1}{7x10^{15}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p1} = 1.23x10^{-6}\ cm$$

From the B-E space-charge region,

$$V_{bi2} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(10^{17}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.933\ V$$

Then

$$x_{p2} = \left\{\frac{2(11.7)\left(8.85x10^{-14}\right)(0.933+2)}{1.6x10^{-19}}\times\left(\frac{10^{19}}{10^{17}}\right)\left(\frac{1}{10^{19}+10^{17}}\right)\right\}^{1/2}$$

or

$$x_{p2} = 1.94x10^{-5}\ cm$$

Now

$$x_B = x_{BO} - x_{p1} - x_{p2} = 1.20 - 0.0123 - 0.194$$

or

$$\underline{x_B = 0.994\ \mu m}$$

10.19

Low injection limit is reached when $p_C(0) = (0.10)N_C$, so that

$$p_C(0) = (0.10)\left(5x10^{14}\right) = 5x10^{13}\ cm^{-3}$$

We have

$$p_{CO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{14}} = 4.5x10^5\ cm^{-3}$$

Also

$$p_C(0) = p_{CO}\exp\left(\frac{V_{CB}}{V_t}\right)$$

or

$$V_{CB} = V_t\ln\left(\frac{p_C(0)}{p_{CO}}\right)$$

$$= (0.0259)\ln\left(\frac{5x10^{13}}{4.5x10^5}\right)$$

or

$$\underline{V_{CB} = 0.48\ V}$$

10.20

(a)

$$\alpha = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$

$$= \frac{1.18}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\alpha = 0.787}$$

(b)

$$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}}$$

$$= \frac{1.20}{1.20 + 0.10} \Rightarrow \underline{\gamma = 0.923}$$

(c)

$$\alpha_T = \frac{J_{nC}}{J_{nE}} = \frac{1.18}{1.20} \Rightarrow \underline{\alpha_T = 0.983}$$

(d)

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}$$

$$= \frac{1.20 + 0.10}{1.20 + 0.20 + 0.10} \Rightarrow \underline{\delta = 0.867}$$

(e)

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.787}{1 - 0.787}$$

or

$$\underline{\beta = 3.69}$$

10.21

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= \left(2.25x10^3\right) \exp\left(\frac{0.50}{0.0259}\right)$$

or

$$\underline{n_B(0) = 5.45x10^{11} \ cm^{-3}}$$

As a good approximation,

$$I_C = \frac{eD_B A n_B(0)}{x_B}$$

$$= \frac{\left(1.6x10^{-19}\right)(20)\left(10^{-3}\right)\left(5.45x10^{11}\right)}{10^{-4}}$$

or

$$\underline{I_C = 17.4 \ \mu A}$$

(b)

Base transport factor

$$\alpha_T = \frac{1}{\cosh\left(x_B / L_B\right)}$$

We find

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(20)\left(10^{-7}\right)} = 1.41x10^{-3} \ cm$$

so that

$$\alpha_T = \frac{1}{\cosh(1/14.1)} \Rightarrow \underline{\alpha_T = 0.9975}$$

Emitter injection efficiency

Assuming $D_E = D_B$, $x_B = x_E$, and $L_E = L_B$; then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} = \frac{1}{1 + \frac{10^{17}}{10^{18}}} \Rightarrow \underline{\gamma = 0.909}$$

Then

$$\alpha = \gamma \alpha_T \delta = (0.909)(0.9975)(1) \Rightarrow$$

$$\underline{\alpha = 0.9067}$$

and

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9067}{1 - 0.9067} \Rightarrow \underline{\beta = 9.72}$$

For $I_E = 1.5 \ mA$,

$$I_C = \alpha I_E = (0.9067)(1.5) \Rightarrow \underline{I_C = 1.36 \ mA}$$

(c)

For $I_B = 2 \ \mu A$,

$$I_C = \beta I_B = (9.72)(2) \Rightarrow \underline{I_C = 19.4 \ \mu A}$$

10.22

(a) We have

$$J_{nE} = \frac{eD_B n_{BO}}{L_B}\left\{\frac{1}{\sinh\left(\frac{x_B}{L_B}\right)} + \frac{\left[\exp\left(\frac{V_{BE}}{V_t}\right) - 1\right]}{\tanh\left(\frac{x_B}{L_B}\right)}\right\}$$

We find that

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{5x10^{16}} = 4.5x10^3 \ cm^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(15)\left(5x10^{-8}\right)} = 8.66x10^{-4} \ cm$$

Then

$$J_{nE}=\frac{(1.6x10^{-19})(15)(4.5x10^{3})}{8.66x10^{-4}}\times\left\{\frac{1}{\sinh\left(\frac{0.70}{8.66}\right)}+\frac{\exp\left(\frac{0.60}{0.0259}\right)}{\tanh\left(\frac{0.70}{8.66}\right)}\right\}$$

or

$$\underline{J_{nE}=1.79\ A/cm^2}$$

We also have

$$J_{pE}=\frac{eD_Ep_{EO}}{L_E}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\cdot\frac{1}{\tanh\left(\frac{x_E}{L_E}\right)}$$

Also

$$p_{EO}=\frac{n_i^2}{N_E}=\frac{(1.5x10^{10})^2}{10^{18}}=2.25x10^2\ cm^{-3}$$

and

$$L_E=\sqrt{D_E\tau_{EO}}=\sqrt{(8)(10^{-8})}=2.83x10^{-4}\ cm$$

Then

$$J_{pE}=\frac{(1.6x10^{-19})(8)(2.25x10^{2})}{2.83x10^{-4}}\times\left[\exp\left(\frac{0.60}{0.0259}\right)-1\right]\cdot\frac{1}{\tanh\left(\frac{0.8}{2.83}\right)}$$

or

$$\underline{J_{pE}=0.0425\ A/cm^2}$$

We can find

$$J_{nC}=\frac{eD_Bn_{BO}}{L_B}\left\{\frac{\left[\exp\left(\frac{0.60}{0.0259}\right)-1\right]}{\sinh\left(\frac{x_B}{L_B}\right)}+\frac{1}{\tanh\left(\frac{x_B}{L_B}\right)}\right\}$$

$$=\frac{(1.6x10^{-19})(15)(4.5x10^{3})}{8.66x10^{-4}}\times\left\{\frac{\exp\left(\frac{0.60}{0.0259}\right)}{\sinh\left(\frac{0.7}{8.66}\right)}+\frac{1}{\tanh\left(\frac{0.7}{8.66}\right)}\right\}$$

or

$$\underline{J_{nC}=1.78\ A/cm^2}$$

The recombination current is

$$J_R=J_{rO}\exp\left(\frac{eV_{BE}}{2kT}\right)$$

$$=(3x10^{-8})\exp\left(\frac{0.60}{2(0.0259)}\right)$$

or

$$\underline{J_R=3.22x10^{-3}\ A/cm^2}$$

(b)
Using the calculated currents, we find

$$\gamma=\frac{J_{nE}}{J_{nE}+J_{pE}}=\frac{1.79}{1.79+0.0425}\Rightarrow$$

$$\underline{\gamma=0.977}$$

We find

$$\alpha_T=\frac{J_{nC}}{J_{nE}}=\frac{1.78}{1.79}\Rightarrow\quad\underline{\alpha_T=0.994}$$

and

$$\delta=\frac{J_{nE}+J_{pE}}{J_{nE}+J_R+J_{pE}}=\frac{1.79+0.0425}{1.79+0.00322+0.0425}$$

or

$$\underline{\delta=0.998}$$

Then

$$\alpha=\gamma\alpha_T\delta=(0.977)(0.994)(0.998)\Rightarrow$$

$$\underline{\alpha=0.969}$$

Now

$$\beta=\frac{\alpha}{1-\alpha}=\frac{0.969}{1-0.969}\Rightarrow\quad\underline{\beta=31.3}$$

10.23

(a) $$\gamma=\frac{1}{1+\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}}\approx1-\frac{N_B}{N_E}\cdot\frac{D_E}{D_B}\cdot\frac{x_B}{x_E}$$

or

$$\gamma\approx1-K\cdot\frac{N_B}{N_E}$$

(i)

$$\frac{\gamma(B)}{\gamma(A)}=\frac{1-\frac{2N_{BO}}{N_E}\cdot K}{1-\frac{N_{BO}}{N_E}\cdot K}$$

$$\approx\left(1-\frac{2N_{BO}}{N_E}\cdot K\right)\left(1+\frac{N_{BO}}{N_E}\cdot K\right)$$

$$\approx1-\frac{2N_{BO}}{N_E}\cdot K+\frac{N_{BO}}{N_E}\cdot K$$

or

$$\frac{\gamma(B)}{\gamma(A)} \approx 1 - \frac{N_{BO}}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)

$$\frac{\gamma(C)}{\gamma(A)} = 1$$

(b) (i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = \frac{\left(1 - \frac{1}{2} \cdot \frac{(x_{BO}/2)}{L_B}\right)^2}{\left(1 - \frac{1}{2} \cdot \frac{x_{BO}}{L_B}\right)^2}$$

$$\approx \frac{\left(1 - \frac{x_{BO}}{2L_B}\right)}{\left(1 - \frac{x_{BO}}{L_B}\right)} \approx \left(1 - \frac{x_{BO}}{2L_B}\right)\left(1 + \frac{x_{BO}}{L_B}\right)$$

$$\approx 1 - \frac{x_{BO}}{2L_B} + \frac{x_{BO}}{L_B}$$

or

$$\frac{\alpha_T(C)}{\alpha_T(A)} \approx 1 + \frac{x_{BO}}{2L_B}$$

(c) Neglect any change in space charge width. Then

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$\approx 1 - \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \frac{K}{J_{sOB}}}{1 - \frac{K}{J_{sOA}}} \approx \left(1 - \frac{K}{J_{sOB}}\right)\left(1 + \frac{K}{J_{sOA}}\right)$$

$$\approx 1 - \frac{K}{J_{sOB}} + \frac{K}{J_{sOA}}$$

Now

$$J_{sO} \propto n_{BO} = \frac{n_i^2}{N_B}$$

so

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{2N_{BO}K}{C} + \frac{N_{BO}K}{C} = 1 - \frac{N_{BO}K}{C}$$

Then

$$\frac{\delta(B)}{\delta(A)} \approx 1 - \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(ii)
We find

$$\frac{\delta(C)}{\delta(A)} \approx 1 + \frac{J_{rO}\exp\left(\frac{-V_{BE}}{2V_t}\right)}{\left(\frac{eD_B n_{BO}}{x_B}\right)}$$

(d)
Device C has the largest β. Base transport factor as well as the recombination factor increases.

10.24

(a)

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} = \frac{1}{1 + K \cdot \frac{N_B}{N_E}}$$

or

$$\gamma \approx 1 - K \cdot \frac{N_B}{N_E}$$

(i) Then

$$\frac{\gamma(B)}{\gamma(A)} = \frac{1 - K \cdot \frac{N_B}{2N_{EO}}}{1 - K \cdot \frac{N_B}{N_{EO}}}$$

$$\approx \left(1 - K \cdot \frac{N_B}{2N_{EO}}\right) \cdot \left(1 + K \cdot \frac{N_B}{N_{EO}}\right)$$

$$\approx 1 - K \cdot \frac{N_B}{2N_{EO}} + K \cdot \frac{N_B}{N_{EO}}$$

or

$$= 1 + K \cdot \frac{N_B}{2N_{EO}}$$

or

$$\frac{\gamma(B)}{\gamma(A)} = 1 + \frac{N_B}{2N_{EO}} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}$$

(ii)
Now

$$\gamma = \frac{1}{1 + K' \cdot \dfrac{x_B}{x_E}} \approx 1 - K' \cdot \frac{x_B}{x_E}$$

Then

$$\frac{\gamma(C)}{\gamma(A)} = \frac{1 - K' \cdot \dfrac{x_B}{(x_{EO}/2)}}{1 - K' \cdot \dfrac{x_B}{x_{EO}}}$$

$$\approx \left(1 - K' \cdot \frac{2x_B}{x_{EO}}\right) \cdot \left(1 + K' \cdot \frac{x_B}{x_{EO}}\right)$$

$$\approx 1 - 2K' \cdot \frac{x_B}{x_{EO}} + K' \cdot \frac{x_B}{x_{EO}}$$

$$= 1 - K' \cdot \frac{x_B}{x_{EO}}$$

or

$$\frac{\gamma(C)}{\gamma(A)} = 1 - \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_{EO}}$$

(b)

$$\alpha_T = 1 - \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2$$

so
(i)

$$\frac{\alpha_T(B)}{\alpha_T(A)} = 1$$

and
(ii)

$$\frac{\alpha_T(C)}{\alpha_T(A)} = 1$$

(c)
Neglect any change in space charge width

$$\delta = \frac{1}{1 + \dfrac{J_{rO}}{J_{SO}} \exp\left(\dfrac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \dfrac{k}{J_{SO}}} \approx 1 - \frac{k}{J_{SO}}$$

(i)

$$\frac{\delta(B)}{\delta(A)} = \frac{1 - \dfrac{k}{J_{SOB}}}{1 - \dfrac{k}{J_{SOA}}} \approx \left(1 - \frac{k}{J_{SOB}}\right)\left(1 + \frac{k}{J_{SOA}}\right)$$

$$\approx 1 - \frac{k}{J_{SOB}} + \frac{k}{J_{SOA}}$$

Now

$$J_{SO} \propto \frac{1}{N_E x_E}$$

so
(i)

$$\frac{\delta(B)}{\delta(A)} = 1 - k'(2N_{EO}) + k'(N_{EO})$$

or

$$\frac{\delta(B)}{\delta(A)} = 1 - k' \cdot (N_{EO})$$

(recombination factor decreases)
(ii)
We have

$$\frac{\delta(C)}{\delta(A)} = 1 - k'' \cdot \left(\frac{x_{EO}}{2}\right) + k'' \cdot (x_{EO})$$

or

$$\frac{\delta(C)}{\delta(A)} = 1 + \frac{1}{2} k'' \cdot x_{EO}$$

(recombination factor increases)

10.25
(b)

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

Then

$$n_B(0) = n_{BO} \exp\left(\frac{V_{BC}}{V_t}\right)$$

$$= (2.25x10^3)\exp\left(\frac{0.6}{0.0259}\right) = 2.59x10^{13} \ cm^{-3}$$

Now

$$J_{nC} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{(1.6x10^{-19})(20)(2.59x10^{13})}{10^{-4}}$$

or

$J_{nC} = 0.829 \ A/cm^2$

Assuming a long collector,

$$J_{pC} = \frac{eD_C p_{nO}}{L_C} \exp\left(\frac{V_{BC}}{V_t}\right)$$

where

$$p_{nO} = \frac{n_i^2}{N_C} = \frac{\left(1.5x10^{10}\right)^2}{10^{16}} = 2.25x10^4 \ cm^{-3}$$

and

$$L_C = \sqrt{D_C \tau_{CO}} = \sqrt{(15)\left(2x10^{-7}\right)} = 1.73x10^{-3} \ cm$$

Then

$$J_{pC} = \frac{\left(1.6x10^{-19}\right)(15)\left(2.25x10^4\right)}{1.73x10^{-3}} \exp\left(\frac{0.6}{0.0259}\right)$$

or

$$J_{pC} = 0.359 \ A/cm^2$$

The collector current is

$$I_C = \left(J_{nC} + J_{pC}\right) \cdot A = (0.829 + 0.359)\left(10^{-3}\right)$$

or

$$\underline{I_C = 1.19 \ mA}$$

The emitter current is

$$I_E = J_{nC} \cdot A = (0.829)\left(10^{-3}\right)$$

or

$$\underline{I_E = 0.829 mA}$$

10.26

(a)

$$\alpha_T = \frac{1}{\cosh\left(x_B/L_B\right)} \qquad \beta = \frac{\alpha_T}{1-\alpha_T}$$

x_B/L_B	α_T	β
0.01	0.99995	19,999
0.10	0.995	199
1.0	0.648	1.84
10.0	0.0000908	≈ 0

(b)

For $D_E = D_B$, $L_E = L_B$, $x_E = x_B$, we have

$$\gamma = \frac{1}{1 + \left(p_{EO}/n_{BO}\right)} = \frac{1}{1 + \left(N_B/N_C\right)}$$

and

$$\beta = \frac{\gamma}{1-\gamma}$$

N_B/N_E	γ	β
0.01	0.990	99
0.10	0.909	9.99
1.0	0.50	1.0
10.0	0.0909	0.10

(c)

For $x_B/L_B < 0.10$, the value of β is unreasonably large, which means that the base transport factor is not the limiting factor. For $x_B/L_B > 1.0$, the value of β is very small, which means that the base transport factor will probably be the limiting factor.

If $N_B/N_E < 0.01$, the emitter injection efficiency is probably not the limiting factor. If, however, $N_B/N_E > 0.01$, then the current gain is small and the emitter injection efficiency is probably the limiting factor.

10.27

We have

$$J_{sO} = \frac{eD_B n_{BO}}{L_B \tanh\left(x_B/L_B\right)}$$

Now

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3 \ cm^{-3}$$

and

$$L_B = \sqrt{D_B \tau_{BO}} = \sqrt{(25)\left(10^{-7}\right)} = 15.8x10^{-4} \ cm$$

Then

$$J_{sO} = \frac{\left(1.6x10^{-19}\right)(25)\left(2.25x10^3\right)}{\left(15.8x10^{-4}\right)\tanh\left(0.7/15.8\right)}$$

or

$$J_{sO} = 1.3x10^{-10} \ A/cm^2$$

Now

$$\delta = \frac{1}{1 + \frac{J_{rO}}{J_{sO}} \cdot \exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

$$= \frac{1}{1 + \frac{2x10^{-9}}{1.3x10^{-10}} \cdot \exp\left(\frac{-V_{BE}}{2(0.0259)}\right)}$$

or

(a)

$$\delta = \frac{1}{1+(15.38)\exp\left(\frac{-V_{BE}}{0.0518}\right)}$$

and
(b)

$$\beta = \frac{\delta}{1-\delta}$$

Now

V_{BE}	δ	β
0.20	0.755	3.08
0.40	0.993	142
0.60	0.99986	7,142

(c)
If $V_{BE} < 0.4\ V$, the recombination factor is likely the limiting factor in the current gain.

10.28

For $\beta = 120 = \frac{\alpha}{1-\alpha} \Rightarrow \alpha = \frac{\beta}{1+\beta}$

So

$$\alpha = \frac{120}{121} = 0.9917$$

Now

$$\alpha = \gamma\alpha_T\delta = 0.9917 = (0.998)x^2$$

where

$$x = \alpha_T = \gamma = 0.9968$$

We have

$$\alpha_T = \frac{1}{\cosh\left(\frac{x_B}{L_B}\right)} = 0.9968$$

which means

$$\frac{x_B}{L_B} = 0.0801$$

We find

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$x_B(\max) = (0.0801)(15.8) \Rightarrow$$

$$\underline{x_B(\max) = 1.26\ \mu m}$$

We also have

$$\gamma = \frac{1}{1+\frac{p_{EO}}{n_{BO}}\cdot\frac{D_E}{D_B}\cdot\frac{L_B}{L_E}\cdot\frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

where

$$L_E = \sqrt{D_E\tau_{EO}} = \sqrt{(10)(5x10^{-8})} = 7.07\ \mu m$$

Then

$$0.9968 = \frac{1}{1+\frac{p_{EO}}{n_{BO}}\cdot\left(\frac{10}{25}\right)\left(\frac{15.8}{7.07}\right)\frac{\tanh(1.26/15.8)}{\tanh(0.5/7.07)}}$$

which yields

$$\frac{p_{EO}}{n_{BO}} = 0.003186 = \frac{N_B}{N_E}$$

Finally

$$N_E = \frac{N_B}{0.003186} = \frac{10^{16}}{0.003186} \Rightarrow$$

$$\underline{N_E = 3.14x10^{18}\ cm^{-3}}$$

10.29

(a) We have $J_{rO} = 5x10^{-8}\ A/cm^2$

We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{5x10^{16}} = 4.5x10^3\ cm^{-3}$$

and

$$L_B = \sqrt{D_B\tau_{BO}} = \sqrt{(25)(10^{-7})} = 15.8\ \mu m$$

Then

$$J_{sO} = \frac{eD_Bn_{BO}}{L_B\tanh(x_B/L_B)}$$

$$= \frac{(1.6x10^{-19})(25)(4.5x10^3)}{(15.8x10^{-4})\tanh(x_B/L_B)}$$

or

$$J_{sO} = \frac{1.14x10^{-11}}{\tanh(x_B/L_B)}$$

We have

$$\delta = \frac{1}{1+\frac{J_{rO}}{J_{sO}}\cdot\exp\left(\frac{-V_{BE}}{2V_t}\right)}$$

For $T = 300K$ and $V_{BE} = 0.55\ V$,

$\delta = 0.995 =$

$$\frac{1}{1+\left(\frac{5x10^{-8}}{1.14x10^{-11}}\right)\cdot\tanh\left(\frac{x_B}{L_B}\right)\cdot\exp\left(\frac{-0.55}{2(0.0259)}\right)}$$

which yields

$$\frac{x_B}{L_B} = 0.047$$

or

$$x_B = (0.047)(15.8x10^{-4}) \Rightarrow$$

$$\underline{x_B = 0.742\ \mu m}$$

(b)

For $T = 400K$ and $J_{rO} = 5x10^{-8}\ A/cm^2$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = \left(\frac{400}{300}\right)^3 \cdot \frac{\exp\left[\frac{-E_g}{(0.0259)(400/300)}\right]}{\exp\left[\frac{-E_g}{(0.0259)}\right]}$$

For $E_g = 1.12\ eV$,

$$\frac{n_{BO}(400)}{n_{BO}(300)} = 1.17x10^5$$

or

$$n_{BO}(400) = (1.17x10^5)(4.5x10^3)$$

$$= 5.27x10^8\ cm^{-3}$$

Then

$$J_{sO} = \frac{(1.6x10^{-19})(25)(5.27x10^8)}{(15.8x10^{-4})\tanh(0.742/15.8)}$$

or

$$J_{sO} = 2.84x10^{-5}\ A/cm^2$$

Finally,

$$\delta = \frac{1}{1 + \frac{5x10^{-8}}{2.84x10^{-5}} \cdot \exp\left[\frac{-0.55}{2(0.0259)(400/300)}\right]}$$

or

$$\underline{\delta = 0.9999994}$$

10.30
Computer plot

10.31
Computer plot

10.32
Computer plot

10.33
Computer plot

10.34
Metallurgical base width $= 1.2\ \mu m = x_B + x_n$

We have

$$p_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{10^{16}} = 2.25x10^4\ cm^{-3}$$

and

$$p_B(0) = p_{BO}\exp\left(\frac{V_{EB}}{V_t}\right)$$

$$= (2.25x10^4)\exp\left(\frac{0.625}{0.0259}\right)$$

$$= 6.8x10^{14}\ cm^{-3}$$

Now

$$J_p = eD_B\frac{dp_B}{dx} = eD_B\left(\frac{p_B(0)}{x_B}\right)$$

$$= \frac{(1.6x10^{-19})(10)(6.8x10^{14})}{x_B}$$

or

$$J_p = \frac{1.09x10^{-3}}{x_B}$$

We have

$$x_n = \left\{\frac{2\in(V_{bi}+V_R)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5x10^{10})^2}\right] = 0.635\ V$$

We can write

$$x_n = \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_R)}{1.6x10^{-19}} \times\left(\frac{10^{15}}{10^{16}}\right)\left(\frac{1}{10^{15}+10^{16}}\right)\right\}^{1/2}$$

or

$$x_n = \left\{(1.177x10^{-10})(V_{bi}+V_R)\right\}^{1/2}$$

We know

$$x_B = 1.2x10^{-4} - x_n$$

For $V_R = V_{BC} = 5\ V$

$$x_n = 0.258x10^{-4}\ cm \Rightarrow x_B = 0.942x10^{-4}\ cm$$

Then

$$\underline{J_p = 11.6\ A/cm^2}$$

For $V_R = V_{BC} = 10\ V$,

$x_n = 0.354x10^{-4}\ cm \Rightarrow x_B = 0.846x10^{-4}\ cm$

Then

$$\underline{J_p = 12.9\ A/cm^2}$$

For $V_R = V_{BC} = 15\,V$,

$x_n = 0.429x10^{-4}\ cm \Rightarrow x_B = 0.771x10^{-4}\ cm$

Then

$$\underline{J_p = 14.1\ A/cm^2}$$

(b)

We can write

$$J_p = g'(V_{EC} + V_A)$$

where

$$g' = \frac{\Delta J_p}{\Delta V_{EC}} = \frac{\Delta J_p}{\Delta V_{BC}} = \frac{14.1 - 11.6}{10}$$

or

$$g' = 0.25\ mA/cm^2/V$$

Now

$J_p = 11.6\ A/cm^2$ at

$$V_{EC} = V_{BC} + V_{EB} = 5 + 0.626 = 5.626\,V$$

Then

$$11.6 = (0.25)(5.625 + V_A)$$

which yields

$$\underline{V_A = 40.8\,V}$$

10.35 We find

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{(1.5x10^{10})^2}{3x10^{16}} = 7.5x10^3\ cm^{-3}$$

and

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$
$$= (7.5x10^3)\exp\left(\frac{0.7}{0.0259}\right)$$

or

$$n_B(0) = 4.10x10^{15}\ cm^{-3}$$

We have

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$
$$= \frac{(1.6x10^{-19})(20)(4.10x10^{15})}{x_B}$$

or

$$J = \frac{1.312x10^{-2}}{x_B}$$

Neglecting the space charge width at the B-E junction, we have

$$x_B = x_{BO} - x_p$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{16})(5x10^{15})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.705\,V$$

and

$$x_p = \left\{\frac{2\in(V_{bi} + V_{CB})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right\}^{1/2}$$
$$= \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi} + V_{CB})}{1.6x10^{-19}}\right.$$
$$\left.\times\left(\frac{5x10^{15}}{3x10^{16}}\right)\left(\frac{1}{5x10^{15} + 3x10^{16}}\right)\right\}^{1/2}$$

or

$$x_p = \left\{(6.163x10^{-11})(V_{bi} + V_{CB})\right\}^{1/2}$$

Now, for $V_{CB} = 5\,V$, $x_p = 0.1875\ \mu m$, and

For $V_{CB} = 10\,V$, $x_p = 0.2569\ \mu m$

(a)

$x_{BO} = 1.0\ \mu m$

For $V_{CB} = 5\,V$, $x_B = 1.0 - 0.1875 = 0.8125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.8125x10^{-4}} = 161.5\ A/cm^2$$

For $V_{CB} = 10\,V$, $x_B = 1.0 - 0.2569 = 0.7431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.7431x10^{-4}} = 176.6\ A/cm^2$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}(V_{CE} + V_A)$$

where

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{176.6 - 161.5}{5}$$
$$= 3.02\ A/cm^2/V$$

Then

$$161.5 = 3.02(5.7 + V_A) \Rightarrow$$
$$\underline{V_A = 47.8\,V}$$

(b)

$x_{BO} = 0.80\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.80 - 0.1875 = 0.6125\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.6125x10^{-4}} = 214.2\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.80 - 0.2569 = 0.5431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.5431x10^{-4}} = 241.6\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{241.6 - 214.2}{5}$$

$$= 5.48\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

or

$$214.2 = 5.48\left(5.7 + V_A\right) \Rightarrow$$

$$\underline{V_A = 33.4\ V}$$

(c)

$x_{BO} = 0.60\ \mu m$

For $V_{CB} = 5\ V$, $x_B = 0.60 - 0.1875 = 0.4124\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.4125x10^{-4}} = 318.1\ A/cm^2$$

For $V_{CB} = 10\ V$, $x_B = 0.60 - 0.2569 = 0.3431\ \mu m$

Then

$$J = \frac{1.312x10^{-2}}{0.3431x10^{-4}} = 382.4\ A/cm^2$$

Now

$$\frac{\Delta J}{\Delta V_{CE}} = \frac{\Delta J}{\Delta V_{CB}} = \frac{382.4 - 318.1}{5}$$

$$= 12.86\ A/cm^2/V$$

We can write

$$J = \frac{\Delta J}{\Delta V_{CE}}\left(V_{CE} + V_A\right)$$

so

$$318.1 = 12.86\left(5.7 + V_A\right) \Rightarrow$$

$$\underline{V_A = 19.0\ V}$$

10.36

Neglect the B-E space charge region

$$n_{BO} = \frac{n_i^2}{N_B} = \frac{\left(1.5x10^{10}\right)^2}{10^{17}} = 2.25x10^3\ cm^{-3}$$

Then

$$n_B(0) = n_{BO}\exp\left(\frac{V_{BE}}{V_t}\right)$$

$$= 2.25x10^3 \exp\left(\frac{0.60}{0.0259}\right) = 2.59x10^{13}\ cm^{-3}$$

$$J = eD_B\frac{dn_B}{dx} = \frac{eD_B n_B(0)}{x_B}$$

$$= \frac{\left(1.6x10^{-19}\right)(20)\left(2.59x10^{13}\right)}{x_B}$$

or

$$J = \frac{8.29x10^{-5}}{x_B}$$

(a)

Now $x_B = x_{BO} - x_p$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{16}\right)\left(10^{17}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.754\ V$$

Also

$$x_p = \left[\frac{2\in\left(V_{bi} + V_{CB}\right)}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C + N_B}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(V_{bi} + V_{CB}\right)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{10^{17}}\right)\left(\frac{1}{10^{16} + 10^{17}}\right)\right]^{1/2}$$

or

$$x_p = \left[\left(1.177x10^{-11}\right)\left(V_{bi} + V_{CB}\right)\right]^{1/2}$$

For $V_{CB} = 1\ V$, $x_p(1) = 4.544x10^{-6}\ cm$

For $V_{CB} = 5\ V$, $x_p(5) = 8.229x10^{-6}\ cm$

Now

$$x_B = x_{BO} - x_p = 1.1x10^{-4} - x_p$$

Then

For $V_{CB} = 1\ V$, $x_B(1) = 1.055\ \mu m$

For $V_{CB} = 5\ V$, $x_B(5) = 1.018\ \mu m$

So

$$\Delta x_B = 1.055 - 1.018 \Rightarrow$$

or

$$\Delta x_B = 0.037\ \mu m$$

(b)
Now

$$J(1) = \frac{8.29x10^{-5}}{1.055x10^{-4}} = 0.7858\ A/cm^2$$

and

$$J(5) = \frac{8.29x10^{-5}}{1.018x10^{-4}} = 0.8143\ A/cm^2$$

and

$$\Delta J = 0.8143 - 0.7858$$

or

$$\Delta J = 0.0285\ A/cm^2$$

10.37

Let $x_E = x_B$, $L_E = L_B$, $D_E = D_B$

Then the emitter injection efficiency is

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{n_{iE}^2}{N_E} \cdot \frac{N_B}{n_{iB}^2}}$$

where $n_{iB}^2 = n_i^2$

For no bandgap narrowing, $n_{iE}^2 = n_i^2$.

With bandgap narrowing, $n_{iE}^2 = n_i^2 \exp\left(\frac{\Delta E_g}{kT}\right)$,

Then

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)}$$

(a)
No bandgap narrowing, so $\Delta E_g = 0$.

$\alpha = \gamma \alpha_T \delta = \gamma (0.995)^2$. We find

N_E	γ	α	β
E17	0.5	0.495	0.980
E18	0.909	0.8999	8.99
E19	0.990	0.980	49
E20	0.9990	0.989	89.9

(b)
Taking into account bandgap narrowing, we find

N_E	$\Delta E_g (meV)$	γ	α	β
E17	0	0.5	0.495	0.98
E18	25	0.792	0.784	3.63
E19	80	0.820	0.812	4.32
E20	230	0.122	0.121	0.14

10.38

(a) We have

$$\gamma = \frac{1}{1 + \frac{p_{EO} D_E L_B}{n_{BO} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For $x_E = x_B$, $L_E = L_B$, $D_E = D_B$, we obtain

$$\gamma = \frac{1}{1 + \frac{p_{EO}}{n_{BO}}} = \frac{1}{1 + \frac{(n_i^2/N_E)\exp(\Delta E_g/kT)}{(n_i^2/N_B)}}$$

or

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E} \exp\left(\frac{\Delta E_g}{kT}\right)}$$

For $N_E = 10^{19}\ cm^{-3}$, we have $\Delta E_g = 80\ meV$.

Then

$$0.996 = \frac{1}{1 + \frac{N_B}{10^{19}} \exp\left(\frac{0.080}{0.0259}\right)}$$

which yields

$$N_B = 1.83x10^{15}\ cm^{-3}$$

(b)
Neglecting bandgap narrowing, we would have

$$\gamma = \frac{1}{1 + \frac{N_B}{N_E}} \Rightarrow 0.996 = \frac{1}{1 + \frac{N_B}{10^{19}}}$$

which yields

$$N_B = 4.02x10^{16}\ cm^{-3}$$

10.39

(a)

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{(S/2)}{e\mu_p N_B (L x_B)}$$

Then

$$R = \frac{4x10^{-4}}{(1.6x10^{-19})(400)(10^{16})(100x10^{-4})(0.7x10^{-4})}$$

or

$$\underline{R = 893\ \Omega}$$

(b)

$$V = IR = (10x10^{-6})(893) \Rightarrow$$

$$\underline{V = 8.93\ mV}$$

(c)
At $x = 0$,

$$n_p(0) = n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)$$

and at $x = \frac{S}{2}$,

$$n_p'(0) = n_{pO} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)$$

Then

$$\frac{n_p'(0)}{n_p(0)} = \frac{n_{pO} \exp\left(\frac{V_{BE} - 0.00893}{V_t}\right)}{n_{pO} \exp\left(\frac{V_{BE}}{V_t}\right)}$$

$$= \exp\left(\frac{-0.00893}{0.0259}\right) = 0.7084$$

or

$$\underline{\frac{n_p'(0)}{n_p(0)} = 70.8\%}$$

10.40
From problem 10.39(c), we have

$$\frac{n_p'(0)}{n_p(0)} = \exp\left(\frac{-V}{V_t}\right)$$

where V is the voltage drop across the $S/2$ length. Now

$$0.90 = \exp\left(\frac{-V}{0.0259}\right)$$

which yields $V = 2.73\ mV$
We have

$$R = \frac{V}{I} = \frac{2.73x10^{-3}}{10x10^{-6}} = 273\ \Omega$$

We can also write

$$R = \frac{S/2}{e\mu_p N_B (Lx_B)}$$

Solving for S, we find

$$S = 2R\mu_p e N_B L x_B$$

$$= 2(273)(400)(1.6x10^{-19})(10^{16}) \times (100x10^{-4})(0.7x10^{-4})$$

or

$$\underline{S = 2.45\ \mu m}$$

10.41
(a)

$$N_B = N_B(0)\exp\left(\frac{-ax}{x_B}\right)$$

where

$$a = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right) > 0$$

and is a constant. In thermal equilibrium

$$J_p = e\mu_p N_B \mathrm{E} - eD_p \frac{dN_B}{dx} = 0$$

so that

$$\mathrm{E} = \frac{D_p}{\mu_p} \cdot \frac{1}{N_B} \cdot \frac{dN_B}{dx} = \left(\frac{kT}{e}\right) \cdot \frac{1}{N_B} \cdot \frac{dN_B}{dx}$$

which becomes

$$\mathrm{E} = \left(\frac{kT}{e}\right) \cdot \frac{1}{N_B} \cdot N_B(0) \cdot \left(\frac{-a}{x_B}\right) \cdot \exp\left(\frac{-ax}{x_B}\right)$$

$$= \left(\frac{kT}{e}\right) \cdot \left(\frac{-a}{x_B}\right) \cdot \frac{1}{N_B} \cdot N_B$$

or

$$\underline{\mathrm{E} = -\left(\frac{a}{x_B}\right)\left(\frac{kT}{e}\right)}$$

which is a constant.
(b)
The electric field is in the negative x-direction which will aid the flow of minority carrier electrons across the base.
(c)

$$J_n = e\mu_n n \mathrm{E} + eD_n \frac{dn}{dx}$$

Assuming no recombination in the base, J_n will be a constant across the base. Then

$$\frac{dn}{dx} + \left(\frac{\mu_n}{D_n}\right) n \mathrm{E} = \frac{J_n}{eD_n} = \frac{dn}{dx} + n\left(\frac{\mathrm{E}}{V_t}\right)$$

where $V_t = \left(\frac{kT}{e}\right)$

The homogeneous solution to the differential equation is found from

$$\frac{dn_H}{dx} + An_H = 0$$

where $A = \dfrac{\mathrm{E}}{V_t}$

The solution is of the form

$$n_H = n_H(0)\exp(-Ax)$$

The particular solution is found from

$$n_P \cdot A = B$$

where $B = \dfrac{J_n}{eD_n}$

The particular solution is then

$$n_P = \frac{B}{A} = \frac{\left(\dfrac{J_n}{eD_n}\right)}{\left(\dfrac{\mathrm{E}}{V_t}\right)} = \frac{J_n V_t}{eD_n \mathrm{E}} = \frac{J_n}{e\mu_n \mathrm{E}}$$

The total solution is then

$$n = \frac{J_n}{e\mu_n \mathrm{E}} + n_H(0)\exp(-Ax)$$

and

$$n(0) = n_{pO}\exp\left(\frac{V_{BE}}{V_t}\right) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right)$$

Then

$$n_H(0) = \frac{n_i^2}{N_B(0)}\exp\left(\frac{V_{BE}}{V_t}\right) - \frac{J_n}{e\mu_n \mathrm{E}}$$

10.42

(a) The basic pn junction breakdown voltage from the figure for $N_C = 5x10^{15}\ cm^{-3}$ is approximately $BV_{CBO} = 90\ V$.

(b)
We have

$$BV_{CEO} = BV_{CBO}\sqrt[n]{1-\alpha}$$

For $n = 3$ and $\alpha = 0.992$, we obtain

$$BV_{CEO} = 90 \cdot \sqrt[3]{1-0.992} = (90)(0.20)$$

or

$$BV_{CEO} = 18\ V$$

(c)
The B-E breakdown voltage, for $N_B = 10^{17}\ cm^{-3}$, is approximately,

$$BV_{BE} = 12\ V$$

10.43

We want $BV_{CEO} = 60\ V$

So then

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[n]{\beta}} \Rightarrow 60 = \frac{BV_{CBO}}{\sqrt[3]{50}}$$

which yields

$$BV_{CBO} = 221\ V$$

For this breakdown voltage, we need

$$N_C \approx 1.5x10^{15}\ cm^{-3}$$

The depletion width into the collector at this voltage is

$$x_C = x_n = \left\{\frac{2\in(V_{bi}+V_{BC})}{e}\left(\frac{N_B}{N_C}\right)\left(\frac{1}{N_B+N_C}\right)\right\}^{1/2}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(1.5x10^{15})(10^{16})}{(1.5x10^{10})^2}\right] = 0.646\ V$$

and $V_{BC} = BV_{CEO} = 60\ V$

so that

$$x_C = \left\{\frac{2(11.7)(8.85x10^{-14})(0.646+60)}{1.6x10^{-19}} \times\left(\frac{10^{16}}{1.5x10^{15}}\right)\left(\frac{1}{10^{16}+1.5x10^{15}}\right)\right\}^{1/2}$$

or

$$x_C = 6.75\ \mu m$$

10.44

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{16})(5x10^{17})}{(1.5x10^{10})^2}\right] = 0.824\ V$$

At punch-through, we have

$$x_B = 0.70x10^{-4} = x_p(V_{BC} = V_{th}) - x_p(V_{BC} = 0)$$

$$= \left\{\frac{2\in(V_{bi}+V_{pt})}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2} - \left\{\frac{2\in V_{bi}}{e}\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_C+N_B}\right)\right\}^{1/2}$$

which can be written as

$$0.70x10^{-4} = \left\{\frac{2(11.7)(8.85x10^{-14})(V_{bi}+V_{pt})}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}}\right)\left(\frac{1}{5x10^{17}+3x10^{16}}\right)\right\}^{1/2}$$
$$-\left\{\frac{2(11.7)(8.85x10^{-14})(0.824)}{1.6x10^{-19}} \times \left(\frac{5x10^{17}}{3x10^{16}}\right)\left(\frac{1}{5x10^{17}+3x10^{16}}\right)\right\}^{1/2}$$

which becomes

$$0.70x10^{-4} = (0.202x10^{-4})\sqrt{V_{bi}+V_{pt}} - (0.183x10^{-4})$$

We obtain

$$V_{bi}+V_{pt} = 19.1\,V$$

or

$$\underline{V_{pt} = 18.3\,V}$$

Considering the junction alone, avalanche breakdown would occur at approximately $BV \approx 25\,V$.

10.45

(a) Neglecting the B-E junction depletion width,

$$V_{pt} = \frac{eW_B^2}{2\in}\cdot\frac{N_B(N_C+N_B)}{N_C}$$
$$= \left\{\frac{(1.6x10^{-19})(0.5x10^{-4})^2}{2(11.7)(8.85x10^{-14})} \cdot \frac{(10^{17})(10^{17}+7x10^{15})}{(7x10^{15})}\right\}$$

or

$$\underline{V_{pt} = 295\,V}$$

However, actual junction breakdown for these doping concentrations is $\approx 70\,V$. So punch-through will not be reached.

10.46

At punch-through,

$$x_O = \left\{\frac{2\in(V_{bi}+V_{pt})}{e}\cdot\left(\frac{N_C}{N_B}\right)\left(\frac{1}{N_B+N_C}\right)\right\}^{1/2}$$

Since $V_{pt} = 25\,V$, we can neglect V_{bi}.

Then we have

$$(0.75x10^{-4}) = \left\{\frac{2(11.7)(8.85x10^{-14})(25)}{1.6x10^{-19}} \times \left(\frac{10^{16}}{N_B}\right)\left(\frac{1}{10^{16}+N_B}\right)\right\}^{1/2}$$

We obtain

$$\underline{N_B = 1.95x10^{16}\ cm^{-3}}$$

10.47

$$V_{CE}(sat) = \left(\frac{kT}{e}\right)\cdot\ln\left[\frac{I_C(1-\alpha_R)+I_B}{\alpha_F I_B-(1-\alpha_F)I_C}\cdot\frac{\alpha_F}{\alpha_R}\right]$$

We can write

$$\exp\left(\frac{V_{CE}(sat)}{0.0259}\right) = \frac{(1)(1-0.2)+I_B}{(0.99)I_B-(1-0.99)(1)}\left(\frac{0.99}{0.20}\right)$$

or

$$\exp\left(\frac{V_{CE}(sat)}{0.0259}\right) = \left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

(a)

For $V_{CE}(sat) = 0.30\,V$, we find

$$\exp\left(\frac{0.30}{0.0259}\right) = 1.0726x10^5 = \left(\frac{0.8+I_B}{0.99I_B-0.01}\right)(4.95)$$

We find

$$\underline{I_B = 0.01014\ mA}$$

(b)

For $V_{CE}(sat) = 0.20\,V$, we find

$$\underline{I_B = 0.0119\ mA}$$

(c)

For $V_{CE}(sat) = 0.10\,V$, we find

$$\underline{I_B = 0.105\ mA}$$

10.48

For an npn in the active mode, we have $V_{BC} < 0$, so that $\exp\left(\frac{V_{BC}}{V_t}\right) \approx 0$.

Now

$$I_E + I_B + I_C = 0 \Rightarrow I_B = -(I_C + I_E)$$

Then we have

$$I_B = -\left\{\alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]+I_{CS}\right\}$$
$$-\left\{-\alpha_R I_{CS} - I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]\right\}$$

or

$$\underline{I_B = (1-\alpha_F)I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]-(1-\alpha_R)I_{CS}}$$

10.49

We can write

$$I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$
$$=\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E$$

Substituting, we find

$$I_C = \alpha_F\left\{\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]-I_E\right\}$$
$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

From the definition of currents, we have $I_E = -I_C$ for the case when $I_B = 0$. Then

$$I_C = \alpha_F\alpha_R I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$
$$+\alpha_F I_C - I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

When a C-E voltage is applied, then the B-C becomes reverse biased, so $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$. Then

$$I_C = -\alpha_F\alpha_R I_{CS} + \alpha_F I_C + I_{CS}$$

We find

$$\underline{I_C = I_{CEO} = \frac{I_{CS}(1-\alpha_F\alpha_R)}{(1-\alpha_F)}}$$

10.50

We have

$$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{V_{BE}}{V_t}\right)-1\right]$$
$$-I_{CS}\left[\exp\left(\frac{V_{BC}}{V_t}\right)-1\right]$$

For $V_{BC} <\approx 0.1\,V$, $\exp\left(\frac{V_{BC}}{V_t}\right)\approx 0$ and

$I_C \approx$ constant. This equation does not include the base width modulation effect.

For $V_{BE} = 0.2\,V$,

$$I_C = (0.98)(10^{-13})\exp\left(\frac{0.2}{0.0250}\right)+5x10^{-13}$$

or

$$\underline{I_C = 2.22x10^{-10}\ A}$$

For $V_{BE} = 0.4\,V$,

$$\underline{I_C = 5x10^{-7}\ A}$$

For $V_{BE} = 0.6\,V$,

$$\underline{I_C = 1.13x10^{-3}\ A}$$

10.51

Computer Plot

10.52

(a)

$$r_\pi' = \left(\frac{kT}{e}\right)\cdot\frac{1}{I_E} = \frac{0.0259}{0.5x10^{-3}} = 51.8\ \Omega$$

So

$$\tau_e = r_\pi' C_{je} = (51.8)(0.8x10^{-12}) \Rightarrow$$

or

$$\underline{\tau_e = 41.4\ ps}$$

Also

$$\tau_b = \frac{x_B^2}{2D_n} = \frac{(0.7x10^{-4})^2}{2(25)} \Rightarrow$$

or

$$\underline{\tau_b = 98\ ps}$$

We have

$$\tau_c = r_c(C_\mu + C_s) = (30)(2)(0.08x10^{-12}) \Rightarrow$$

or

$$\underline{\tau_c = 4.8\ ps}$$

Also

$$\tau_d = \frac{x_{dc}}{v_s} = \frac{2x10^{-4}}{10^{+7}} \Rightarrow$$

or

$$\underline{\tau_d = 20\ ps}$$

(b)

$$\tau_{ec} = \tau_e + \tau_b + \tau_c + \tau_d$$
$$= 41.4 + 98 + 4.8 + 20 \Rightarrow$$

or

$$\tau_{ec} = 164.2 \ ps$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(164.2x10^{-12}\right)} \Rightarrow$$

or

$$f_T = 970 \ MHz$$

Also

$$f_\beta = \frac{f_T}{\beta} = \frac{970}{50} \Rightarrow$$

or

$$f_\beta = 19.4 \ MHz$$

10.53

$$\tau_b = \frac{x_B^2}{2D_B} = \frac{\left(0.5x10^{-4}\right)^2}{2(20)} = 6.25x10^{-11} \ s$$

We have $\tau_b = 0.2\tau_{ec}$,
So that

$$\tau_{ec} = 3.125x10^{-10} \ s$$

Then

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(3.125x10^{-10}\right)} \Rightarrow$$

or

$$f_T = 509 \ MHz$$

10.54

We have

$$\tau_{ec} = \tau_e + \tau_b + \tau_d + \tau_c$$

We are given

$\tau_b = 100 \ ps$ and $\tau_e = 25 \ ps$

We find

$$\tau_d = \frac{x_d}{v_s} = \frac{1.2x10^{-4}}{10^7} = 1.2x10^{-11} \ s$$

or

$$\tau_d = 12 \ ps$$

Also

$$\tau_c = r_c C_c = (10)\left(0.1x10^{-12}\right) = 10^{-12} \ s$$

or

$$\tau_c = 1 \ ps$$

Then

$$\tau_{ec} = 25 + 100 + 12 + 1 = 138 \ ps$$

We obtain

$$f_T = \frac{1}{2\pi\tau_{ec}} = \frac{1}{2\pi\left(138x10^{-12}\right)} = 1.15x10^9 \ Hz$$

or

$$f_T = 1.15 \ GHz$$

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