Chapter 12

Problem Solutions

12.1
(a)

$$I_D = 10^{-15}\exp\left[\frac{V_{GS}}{(2.1)V_t}\right]$$

For $V_{GS} = 0.5\,V$,

$$I_D = 10^{-15}\exp\left[\frac{0.5}{(2.1)(0.0259)}\right] \Rightarrow$$

$$\underline{I_D = 9.83x10^{-12}\ A}$$

For $V_{GS} = 0.7\,V$,

$$\underline{I_D = 3.88x10^{-10}\ A}$$

For $V_{GS} = 0.9\,V$,

$$\underline{I_D = 1.54x10^{-8}\ A}$$

Then the total current is:

$$I_{Total} = I_D\left(10^6\right)$$

For $V_{GS} = 0.5\,V$, $\underline{I_{Total} = 9.83\ \mu A}$

For $V_{GS} = 0.7\,V$, $\underline{I_{Total} = 0.388\ mA}$

For $V_{GS} = 0.9\,V$, $\underline{I_{Total} = 15.4\ mA}$

(b)
Power: $P = I_{Total}\cdot V_{DD}$
Then
For $V_{GS} = 0.5\,V$, $\underline{P = 49.2\ \mu W}$

For $V_{GS} = 0.7\,V$, $\underline{P = 1.94\ mW}$

For $V_{GS} = 0.9\,V$, $\underline{P = 77\ mW}$

12.2
We have

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

where

$$\phi_{fp} = V_t\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.347\,V$$

We find

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

$$= 0.360\ \mu m/V^{1/2}$$

We have

$$V_{DS}(sat) = V_{GS} - V_T$$

(a)
For $V_{GS} = 5\,V \Rightarrow V_{DS}(sat) = 4.25\,V$
Then

$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+4.25}\right]$$

or

$$\Delta L = 0.0606\ \mu m$$

If ΔL is 10%of L, then $\underline{L = 0.606\ \mu m}$

(b)
For $V_{DS} = 5\,V$, $V_{GS} = 2\,V \Rightarrow V_{DS}(sat) = 1.25\,V$
Then

$$\Delta L = 0.360\left[\sqrt{0.347+5} - \sqrt{0.347+1.25}\right]$$

or

$$\Delta L = 0.377\ \mu m$$

Now if ΔL is 10% of L, then $\underline{L = 3.77\ \mu m}$

12.3

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}(sat) + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

where

$$\phi_{fp} = V_t\ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{4x10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.383\,V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.383)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT} = 0.157\ \mu m$$

Then

$$\left|Q'_{SD}(\max)\right| = eN_a x_{dT}$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(0.157x10^{-4}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 10^{-7}\ C/cm^2$$

Now

$$V_T = \left(\left|Q'_{SD}(\max)\right| - Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

so that

$$V_T = \frac{\left[10^{-7} - \left(1.6x10^{-19}\right)\left(3x10^{10}\right)\right]\left(400x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)} + 0 + 2(0.383)$$

or

$$V_T = 1.87\ V$$

Now

$$V_{DS}(sat) = V_{GS} - V_T = 5 - 1.87 = 3.13\ V$$

We find

$$\sqrt{\frac{2\in}{eN_a}} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2} = 1.80x10^{-5}$$

Now

$$\Delta L = 1.80x10^{-5}\cdot\left[\sqrt{0.383 + 3.13 + \Delta V_{DS}} - \sqrt{0.383 + 3.13}\right]$$

or

$$\Delta L = 1.80x10^{-5}\left[\sqrt{3.513 + \Delta V_{DS}} - \sqrt{3.513}\right]$$

We obtain

ΔV_{DS}	$\Delta L(\mu m)$
0	0
1	0.0451
2	0.0853
3	0.122
4	0.156
5	0.188

12.4
Computer plot

12.5
Plot

12.6
Plot

12.7

(a) Assume $V_{DS}(sat) = 1\ V$, We have

$$E_{sat} = \frac{V_{DS}(sat)}{L}$$

We find

$L(\mu m)$	$E_{sat}(V/cm)$
3	$3.33x10^3$
1	10^4
0.5	$2x10^4$
0.25	$4x10^4$
0.13	$7.69x10^4$

(b)

Assume $\mu_n = 500\ cm^2/V-s$, we have

$$v = \mu_n E_{sat}$$

Then

For $L = 3\ \mu m$, $\underline{v = 1.67x10^6\ cm/s}$

For $L = 1\ \mu m$, $\underline{v = 5x10^6\ cm/s}$

For $L \le 0.5\ \mu m$, $\underline{v \approx 10^7\ cm/s}$

12.8

We have $I'_D = L(L - \Delta L)^{-1} I_D$

We may write

$$g_O = \frac{\partial I'_D}{\partial V_{DS}} = (-1)L(L - \Delta L)^{-2} I_D\left(\frac{-\partial(\Delta L)}{\partial V_{DS}}\right)$$

$$= \frac{L}{(L - \Delta L)^2}\cdot I_D\cdot\frac{\partial(\Delta L)}{\partial V_{DS}}$$

We have

$$\Delta L = \sqrt{\frac{2\in}{eN_a}}\cdot\left[\sqrt{\phi_{fp} + V_{DS}} - \sqrt{\phi_{fp} + V_{DS}(sat)}\right]$$

We find

$$\frac{\partial(\Delta L)}{\partial V_{DS}} = \sqrt{\frac{2\in}{eN_a}}\cdot\frac{1}{2\sqrt{\phi_{fp} + V_{DS}}}$$

(a)

For $V_{GS} = 2\ V$, $\Delta V_{DS} = 1\ V$, and

$$V_{DS}(sat) = V_{GS} - V_T = 2 - 0.8 = 1.2\ V$$

Also

$$V_{DS} = V_{DS}(sat) + \Delta V_{DS} = 1.2 + 1 = 2.2\ V$$

and

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

Now

$$\sqrt{\frac{2\in}{eN_a}}=\left[\frac{2(11.7)\left(8.85x10^{-14}\right)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

$$=0.2077\ \mu m/V^{1/2}$$

We find

$$\Delta L=0.2077\left[\sqrt{0.376+2.2}-\sqrt{0.376+1.2}\right]$$

$$=0.0726\ \mu m$$

Then

$$\frac{\partial(\Delta L)}{\partial V_{DS}}=\frac{0.2077}{2}\cdot\frac{1}{\sqrt{0.376+2.2}}$$

$$=0.0647\ \mu m/V$$

From the previous problem,

$I_D=0.48\ mA$, $L=2\ \mu m$

Then

$$g_O=\frac{2}{(2-0.0726)^2}\left(0.48x10^{-3}\right)(0.0647)$$

or

$$g_O=1.67x10^{-5}\ S$$

so that

$$r_O=\frac{1}{g_O}=59.8\ k\Omega$$

(b)

If $L=1\ \mu m$, then from the previous problem, we would have $I_D=0.96\ mA$, so that

$$g_O=\frac{1}{(1-0.0726)^2}\left(0.96x10^{-3}\right)(0.0647)$$

or

$$g_O=7.22x10^{-5}\ S$$

so that

$$r_O=\frac{1}{g_O}=13.8\ k\Omega$$

12.9

(a)

$$I_D(sat)=\frac{W\mu_n C_{ox}}{2L}\left(V_{GS}-V_T\right)^2$$

$$=\left(\frac{10}{2}\right)(500)\left(6.9x10^{-8}\right)\left(V_{GS}-1\right)^2$$

or

$$I_D(sat)=0.173\left(V_{GS}-1\right)^2\ (mA)$$

and

$$\sqrt{I_D(sat)}=\sqrt{0.173}\left(V_{GS}-1\right)\ (mA)^{1/2}$$

(b)

Let $\mu_{eff}=\mu_O\left(\frac{E_{eff}}{E_C}\right)^{-1/3}$

Where $\mu_O=1000\ cm^2/V-s$ and $E_C=2.5x10^4\ V/cm$.

Let $E_{eff}=\frac{V_{GS}}{t_{ox}}$

We find

$$C_{ox}=\frac{\in_{ox}}{t_{ox}}\Rightarrow t_{ox}=\frac{\in_{ox}}{C_{ox}}=\frac{(3.9)\left(8.85x10^{-14}\right)}{6.9x10^{-8}}$$

or

$$t_{ox}=500\ \mathring{A}$$

Then

V_{GS}	E_{eff}	μ_{eff}	$\sqrt{I_D(sat)}$
1	--	--	0
2	4E5	397	0.370
3	6E5	347	0.692
4	8E5	315	0.989
5	10E5	292	1.27

(c)

The slope of the variable mobility curve is not constant, but is continually decreasing.

12.10

Plot

12.11

$$V_T=V_{FB}+\frac{\left|Q'_{SD}(\max)\right|}{C_{ox}}+2\phi_{fp}$$

We find

$$\phi_{fp}=V_t\ln\left(\frac{N_a}{n_i}\right)=(0.0259)\ln\left(\frac{5x10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp}=0.389\ V$$

and

$$x_{dT}=\left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$=\left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.389)}{\left(1.6x10^{-19}\right)\left(5x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dT}=0.142\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$
$$= (1.6x10^{-19})(5x10^{16})(0.142x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.14x10^{-7}\ C/cm^2$$

Also

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{400x10^{-8}} = 8.63x10^{-8}\ F/cm^2$$

Then

$$V_T = -1.12 + \frac{1.14x10^{-7}}{8.63x10^{-8}} + 2(0.389)$$

or

$$\underline{V_T = +0.90\ V}$$

(a)

$$I_D = \frac{W\mu_n C_{ox}}{2L}\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

and

$$V_{DS}(sat) = V_{GS} - V_T$$

We have

$$I_D = \left(\frac{20}{2}\right)\left(\frac{1}{2}\right)(400)(8.63x10^{-8}) \times\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

or

$$I_D = 0.173\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right](mA)$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 1\ V$,

$$\underline{I_D(sat) = 0.173\ mA}$$

For $V_{DS} = V_{DS}(sat) = V_{GS} - V_T = 2\ V$,

$$\underline{I_D(sat) = 0.692\ mA}$$

(b)

For $V_{DS} \le 1.25\ V$, $\mu = \mu_n = 400\ cm^2/V-s$. The curve for $V_{GS} - V_T = 1\ V$ is unchanged. For $V_{GS} - V_T = 2\ V$ and $0 \le V_{DS} \le 1.25\ V$, the curve in unchanged. For $V_{DS} \ge 1.25\ V$, the current is constant at

$$I_D = 0.173\left[2(2)(1.25) - (1.25)^2\right] = 0.595\ mA$$

When velocity saturation occurs, $V_{DS}(sat) = 1.25\ V$ for the case of $V_{GS} - V_T = 2\ V$.

12.12

Plot

12.13

(a) Non-saturation region

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2(V_{GS}-V_T)V_{DS} - V_{DS}^2\right]$$

We have

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \Rightarrow \frac{C_{ox}}{k}$$

and

$$W \Rightarrow kW,\ L \Rightarrow kL$$

also

$$V_{GS} \Rightarrow kV_{GS},\ V_{DS} \Rightarrow kV_{DS}$$

So

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[2(kV_{GS}-V_T)kV_{DS} - (kV_{DS})^2\right]$$

Then

$$\underline{I_D \Rightarrow \approx kI_D}$$

In the saturation region

$$I_D = \frac{1}{2}\mu_n\left(\frac{C_{ox}}{k}\right)\left(\frac{kW}{kL}\right)\left[kV_{GS}-V_T\right]^2$$

Then

$$\underline{I_D \Rightarrow \approx kI_D}$$

(b)

$$\underline{P = I_D V_{DD} \Rightarrow (kI_D)(kV_{DD}) \Rightarrow k^2 P}$$

12.14

$$I_D(sat) = WC_{ox}(V_{GS}-V_T)v_{sat}$$
$$\Rightarrow (kW)\left(\frac{C_{ox}}{k}\right)(kV_{GS}-V_T)v_{sat}$$

or

$$\underline{I_D(sat) \approx kI_D(sat)}$$

12.15

(a)

(i) $I_D = K_n(V_{GS}-V_T)^2 = (0.1)(5-0.8)^2$

or

$$\underline{I_D = 1.764\ mA}$$

(ii)

$$I_D = \left(\frac{0.1}{0.6}\right)\left[(0.6)(5) - 0.8\right]^2$$

or

$$\underline{I_D = 0.807\ mA}$$

(b)

(i) $P = (1.764)(5) \Rightarrow \underline{P = 8.82\ mW}$

(ii) $P = (0.807)(0.6)(5) \Rightarrow \underline{P = 2.42\ mW}$

(c)

Current: Ratio $= \dfrac{0.807}{1.764} = 0.457$

Power: Ratio $= \dfrac{2.42}{8.82} = 0.274$

12.16

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = V_t \ln\left(\frac{N_a}{n_i}\right) = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right)$$

or

$$\phi_{fp} = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

$$= 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left\{\frac{0.3}{1}\left[\sqrt{1+\frac{2(0.3)}{0.3}}-1\right]\right\}$$

or

$$\Delta V_T = -0.137\ V$$

12.17

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1+\frac{2x_{dT}}{r_j}}-1\right]\right\}$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -0.20 = -\frac{(1.6x10^{19})(3x10^{16})(0.18x10^{-4})}{4.31x10^{-8}} \times\left\{\frac{0.6}{L}\left[\sqrt{1+\frac{2(0.18)}{0.6}}-1\right]\right\}$$

or

$$= -0.20 = -\frac{0.319}{L}$$

which yields

$$L = 1.59\ \mu m$$

12.18

We have

$$L' = L-(a+b)$$

and from the geometry

(1) $\left(a+r_j\right)^2 + x_{dT}^2 = \left(r_j + x_{dS}\right)^2$

and

(2) $\left(b+r_j\right)^2 + x_{dT}^2 = \left(r_j + x_{dD}\right)^2$

From (1),

$$\left(a+r_j\right)^2 = \left(r_j + x_{dS}\right)^2 - x_{dT}^2$$

so that

$$a = \sqrt{\left(r_j + x_{dS}\right)^2 - x_{dT}^2} - r_j$$

which can be written as

$$a = r_j\left[\sqrt{\left(1+\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

or

$$a = r_j\left[\sqrt{1 + \frac{2x_{dS}}{r_j} + \left(\frac{x_{dS}}{r_j}\right)^2 - \left(\frac{x_{dT}}{r_j}\right)^2} - 1\right]$$

Define

$$\alpha^2 = \frac{x_{dS}^2 - x_{dT}^2}{r_j^2}$$

We can then write

$$a = r_j\left[\sqrt{1 + \frac{2x_{dS}}{r_j} + \alpha^2} - 1\right]$$

Similarly from (2), we will have

$$b = r_j\left[\sqrt{1 + \frac{2x_{dD}}{r_j} + \beta^2} - 1\right]$$

where

$$\beta^2 = \frac{x_{dD}^2 - x_{dT}^2}{r_j^2}$$

The average bulk charge in the trapezoid (per unit area) is

$$|Q'_B| \cdot L = eN_a x_{dT}\left(\frac{L + L'}{2}\right)$$

or

$$|Q'_B| = eN_a x_{dT}\left(\frac{L + L'}{2L}\right)$$

We can write

$$\frac{L + L'}{2L} = \frac{1}{2} + \frac{L'}{2L} = \frac{1}{2} + \frac{1}{2L}\left[L - (a + b)\right]$$

which is

$$= 1 - \frac{(a + b)}{2L}$$

Then

$$|Q'_B| = eN_a x_{dT}\left[1 - \frac{(a + b)}{2L}\right]$$

Now $|Q'_B|$ replaces $|Q'_{SD}(\max)|$ in the threshold equation. Then

$$\Delta V_T = \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

$$= \frac{eN_a x_{dT}}{C_{ox}}\left[1 - \frac{(a + b)}{2L}\right] - \frac{eN_a x_{dT}}{C_{ox}}$$

or

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{(a + b)}{2L}$$

Then substituting, we obtain

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}} \cdot \frac{r_j}{2L}\left\{\left[\sqrt{1 + \frac{2x_{dS}}{r_j} + \alpha^2} - 1\right] + \left[\sqrt{1 + \frac{2x_{dD}}{r_j} + \beta^2} - 1\right]\right\}$$

Note that if $x_{dS} = x_{dD} = x_{dT}$, then $\alpha = \beta = 0$ and the expression for ΔV_T reduces to that given in the text.

12.19

We have $L' = 0$, so Equation (12.25) becomes

$$\frac{L + L'}{2L} \Rightarrow \frac{L}{2L} = \frac{1}{2} = \left\{1 - \frac{r_j}{L}\left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right]\right\}$$

or

$$\frac{r_j}{L}\left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right] = \frac{1}{2}$$

Then Equation (12.26) is

$$|Q'_B| = eN_a x_{dT}\left(\frac{1}{2}\right)$$

The change in the threshold voltage is

$$\Delta V_T = \frac{|Q'_B|}{C_{ox}} - \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

or

$$\Delta V_T = \frac{(1/2)(eN_a x_{dT})}{C_{ox}} - \frac{(eN_a x_{dT})}{C_{ox}}$$

or

$$\Delta V_T = -\left(\frac{1}{2}\right)\frac{(eN_a x_{dT})}{C_{ox}}$$

12.20

Computer plot

12.21

Computer plot

12.22

$$\Delta V_T = -\frac{eN_a x_{dT}}{C_{ox}}\left\{\frac{r_j}{L}\left[\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right]\right\}$$

$$\Rightarrow -\frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left\{\frac{kr_j}{kL}\left[\sqrt{1+\frac{2kx_{dT}}{kr_j}}-1\right]\right\}$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.23

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}}$$

or

$$C_{ox} = 7.67x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = \frac{(1.6x10^{-19})(10^{16})(0.3x10^{-4})}{7.67x10^{-8}} \times\left[\frac{(\pi/2)(0.3x10^{-4})}{2.5x10^{-4}}\right]$$

or

$$\underline{\Delta V_T = +0.118\ V}$$

12.24

Additional bulk charge due to the ends:

$$\Delta Q_B = eN_a L\left(\frac{1}{2}x_{dT}^2\right)\cdot 2 = eN_a Lx_{dT}(\xi x_{dT})$$

where $\xi = 1$.

Then

$$\Delta V_T = \frac{eN_a x_{dT}^2}{C_{ox}W}$$

We find

$$\phi_{fp} = (0.0259)\ln\left(\frac{3x10^{16}}{1.5x10^{10}}\right) = 0.376\ V$$

and

$$x_{dT} = \left[\frac{4\in\phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.376)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.180\ \mu m$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{800x10^{-8}}$$

or

$$C_{ox} = 4.31x10^{-8}\ F/cm^2$$

Now, we can write

$$W = \frac{eN_a x_{dT}^2}{C_{ox}(\Delta V_T)}$$

$$= \frac{(1.6x10^{-19})(3x10^{16})(0.18x10^{-4})^2}{(4.31x10^{-8})(0.25)}$$

or

$$\underline{W = 1.44\ \mu m}$$

12.25

Computer plot

12.26

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{ox}}\left(\frac{\xi x_{dT}}{W}\right)$$

Assume that ξ is a constant

$$\Rightarrow \frac{e\left(\frac{N_a}{k}\right)(kx_{dT})}{\left(\frac{C_{ox}}{k}\right)}\left(\frac{\xi\cdot kx_{dT}}{kW}\right)$$

or

$$\underline{\Delta V_T = k\Delta V_T}$$

12.27

(a)

$$V_{BD} = (6x10^6)t_{ox} = (6x10^6)(250x10^{-8})$$

or

$$\underline{V_{BD} = 15\ V}$$

(b)
With a safety factor of 3,

$$V_{BD} = \frac{1}{3} \cdot 15 \Rightarrow \underline{V_{BD} = 5\ V}$$

12.28
We want $V_G = 20\ V$. With a safety factor of 3, then $V_{BD} = 60\ V$, so that

$$60 = \left(6x10^{6}\right)t_{ox} \Rightarrow \underline{t_{ox} = 1000\ \mathring{A}}$$

12.29
Snapback breakdown means $\alpha M = 1$, where

$$\alpha = (0.18)\log_{10}\left(\frac{I_D}{3x10^{-9}}\right)$$

and

$$M = \frac{1}{1 - \left(\frac{V_{CE}}{V_{BD}}\right)^m}$$

Let $V_{BD} = 15\ V$, $m = 3$. Now when

$$\alpha M = 1 = \frac{\alpha}{1 - \left(\frac{V_{CE}}{15}\right)^3}$$

we can write this as

$$1 - \left(\frac{V_{CE}}{15}\right)^3 = \alpha \Rightarrow V_{CE} = 15\sqrt[3]{1-\alpha}$$

Now

I_D	α	V_{CE}
E-8	0.0941	14.5
E-7	0.274	13.5
E-6	0.454	12.3
E-5	0.634	10.7
E-4	0.814	8.6
E-3	0.994	2.7

12.30
One Debye length is

$$L_D = \left[\frac{\in (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)\left(8.85x10^{-14}\right)(0.0259)}{\left(1.6x10^{-19}\right)\left(10^{16}\right)}\right]^{1/2}$$

or

$$L_D = 4.09x10^{-6}\ cm$$

Six Debye lengths:

$$6\left(4.09x10^{-6}\right) = 0.246\ \mu m$$

From Example 12.4, we have $x_{dO} = 0.336\ \mu m$, which is the zero-biased source-substrate junction width.
At near punch-through, we will have

$$x_{dO} + 6L_D + x_d = L$$

where x_d is the reverse-biased drain-substrate junction width. Now

$$0.336 + 0.246 + x_d = 1.2 \Rightarrow x_d = 0.618\ \mu m$$ at near punch-through.
We have

$$x_d = \left[\frac{2 \in \left(V_{bi} + V_{DS}\right)}{eN_a}\right]^{1/2}$$

or

$$V_{bi} + V_{DS} = \frac{x_d^2 eN_a}{2 \in}$$

$$= \frac{\left(0.618x10^{-4}\right)^2\left(1.6x10^{-19}\right)\left(10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

which yields

$$V_{bi} + V_{DS} = 2.95\ V$$

From Example 12.4, we have $V_{bi} = 0.874\ V$, so that

$$\underline{V_{DS} = 2.08\ V}$$

which is the near punch-through voltage. The ideal punch-through voltage was

$$\underline{V_{DS} = 4.9\ V}$$

12.31

$$V_{bi} = (0.0259)\ln\left[\frac{\left(10^{19}\right)\left(3x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right] = 0.902\ V$$

The zero-biased source-substrate junction width:

$$x_{dO} = \left[\frac{2 \in V_{bi}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.902)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_{dO} = 0.197\ \mu m$$

The Debye length is

$$L_D = \left[\frac{\in (kT/e)}{eN_a}\right]^{1/2}$$

$$= \left[\frac{(11.7)(8.85x10^{-14})(0.0259)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$L_D = 2.36x10^{-6}\ cm$$

so that

$$6L_D = 6(2.36x10^{-6}) = 0.142\ \mu m$$

Now

$$x_{dO} + 6L_D + x_d = L$$

We have for $V_{DS} = 5\ V$,

$$x_d = \left[\frac{2 \in (V_{bi} + V_{DS})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902 + 5)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.505\ \mu m$$

Then

$$L = 0.197 + 0.142 + 0.505$$

or

$$\underline{L = 0.844\ \mu m}$$

12.32

With a source-to-substrate voltage of 2 volts,

$$x_{dO} = \left[\frac{2 \in (V_{bi} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902 + 2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_{dO} = 0.354\ \mu m$$

We have $6L_D = 0.142\ \mu m$ from the previous problem.

Now

$$x_d = \left[\frac{2 \in (V_{bi} + V_{DS} + V_{SB})}{eN_a}\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.902 + 5 + 2)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$x_d = 0.584\ \mu m$$

Then

$$L = x_{dO} + 6L_D + x_d$$

$$= 0.354 + 0.142 + 0.584$$

or

$$\underline{L = 1.08\ \mu m}$$

12.33

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{2x10^{15}}{1.5x10^{10}}\right) = 0.306\ V$

and

$$\phi_{ms} = -\left(\frac{E_g}{2e} + \phi_{fp}\right) = -\left(\frac{1.12}{2} + 0.306\right)$$

or

$$\phi_{ms} = -0.866\ V$$

Also

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.306)}{(1.6x10^{-19})(2x10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.629\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(2x10^{15})(0.629x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 2.01x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = (2x10^{11})(1.6x10^{-19}) = 3.2x10^{-8}\ C/cm^2$$

Then

$$V_T = \left(|Q'_{SD}(\max)| - Q'_{SS}\right)\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} + 2\phi_{fp}$$

$$= \frac{(2.01x10^{-8} - 3.2x10^{-8})(650x10^{-8})}{(3.9)(8.85x10^{-14})}$$

$$-0.866 + 2(0.306)$$

which yields

$$\underline{V_T = -0.478\ V}$$

(b) We need a shift in threshold voltage in the positive direction, which means we must add acceptor atoms. We need

$$\Delta V_T = +0.80 - (-0.478) = 1.28\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.28)(3.9)(8.85x10^{-14})}{(1.6x10^{-19})(650x10^{-8})}$$

or

$$\underline{D_I = 4.25x10^{11}\ cm^{-2}}$$

12.34

(a) $\phi_{fn} = (0.0259)\ln\left(\frac{10^{16}}{1.5x10^{10}}\right) = 0.347\ V$

and

$$\phi_{ms} = \phi'_{ms} - \left(\chi' + \frac{E_g}{2e} - \phi_{fn}\right)$$

$$= 3.2 - (3.25 + 0.56 - 0.347)$$

or

$$\phi_{ms} = -0.263\ V$$

Also

$$x_{dT} = \left[\frac{4 \in \phi_{fn}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.347)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$$x_{dT} = 0.30\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_d x_{dT}$$

$$= (1.6x10^{-19})(10^{16})(0.30x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 4.8x10^{-8}\ C/cm^2$$

We have

$$Q'_{SS} = (5x10^{11})(1.6x10^{-19}) = 8x10^{-8}\ C/cm^2$$

Now

$$V_T = -(|Q'_{SD}(\max)| + Q'_{SS})\left(\frac{t_{ox}}{\in_{ox}}\right) + \phi_{ms} - 2\phi_{fn}$$

$$= \frac{-(4.8x10^{-8} + 8x10^{-8})(750x10^{-8})}{(3.9)(8.85x10^{-14})}$$

$$-0.263 - 2(0.347)$$

which becomes

$$\underline{V_T = -3.74\ V}$$

(b)

We want $V_T = -0.50\ V$. Need to shift V_T in the positive direction which means we need to add acceptor atoms.

So

$$\Delta V_T = -0.50 - (-3.74) = 3.24\ V$$

Now

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(3.24)(3.9)(8.85x10^{-14})}{(1.6x10^{-19})(750x10^{-8})}$$

or

$$\underline{D_I = 9.32x10^{11}\ cm^{-2}}$$

12.35

(a) $\phi_{fp} = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right) = 0.288\ V$

and

$$x_{dT} = \left[\frac{4 \in \phi_{fp}}{eN_a}\right]^{1/2}$$

$$= \left[\frac{4(11.7)(8.85x10^{-14})(0.288)}{(1.6x10^{-19})(10^{15})}\right]^{1/2}$$

or

$$x_{dT} = 0.863\ \mu m$$

Now

$$|Q'_{SD}(\max)| = eN_a x_{dT}$$

$$= (1.6x10^{-19})(10^{15})(0.863x10^{-4})$$

or

$$|Q'_{SD}(\max)| = 1.38x10^{-8}\ C/cm^2$$

Also

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$V_T = V_{FB} + 2\phi_{fp} + \frac{|Q'_{SD}(\max)|}{C_{ox}}$$

$$= -1.50 + 2(0.288) + \frac{1.38x10^{-8}}{4.6x10^{-8}}$$

or

$$\underline{V_T = -0.624\ V}$$

(b)

Want $V_T = +0.90\ V$, which is a positive shift and we must add acceptor atoms.

$$\Delta V_T = 0.90 - (-0.624) = 1.52\ V$$

Then

$$D_I = \frac{(\Delta V_T)C_{ox}}{e} = \frac{(1.52)(4.6x10^{-8})}{1.6x10^{-19}}$$

or

$$D_I = 4.37x10^{11}\ cm^{-2}$$

(c)
With an applied substrate voltage,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

$$= \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{15}\right)\right]^{1/2}}{4.6x10^{-8}}$$

$$\times\left[\sqrt{2(0.288)+2} - \sqrt{2(0.288)}\right]$$

or

$$\Delta V_T = +0.335\ V$$

Then the threshold voltage is

$$V_T = +0.90 + 0.335$$

or

$$V_T = 1.24\ V$$

12.36

The total space charge width is greater than x_I, so from chapter 11,

$$\Delta V_T = \frac{\sqrt{2e \in N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

Now

$$\phi_{fp} = (0.0259)\ln\left(\frac{10^{14}}{1.5x10^{10}}\right) = 0.228\ V$$

and

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{500x10^{-8}}$$

or

$$C_{ox} = 6.90x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = \frac{\left[2\left(1.6x10^{-19}\right)(11.7)\left(8.85x10^{-14}\right)\left(10^{14}\right)\right]^{1/2}}{6.90x10^{-8}}$$

$$\times\left[\sqrt{2(0.228)+V_{SB}} - \sqrt{2(0.228)}\right]$$

or

$$\Delta V_T = 0.0834\left[\sqrt{0.456+V_{SB}} - \sqrt{0.456}\right]$$

Then

$V_{SB}(V)$	$\Delta V_T(V)$
1	0.0443
3	0.0987
5	0.399

11.37

(a) $\phi_{fn} = (0.0259)\ln\left(\frac{10^{17}}{1.5x10^{10}}\right) = 0.407\ V$

and

$$x_{dT} = \left[\frac{4(11.7)\left(8.85x10^{-14}\right)(0.407)}{\left(1.6x10^{-19}\right)\left(10^{17}\right)}\right]^{1/2}$$

or

$$x_{dT} = 1.026x10^{-5}\ cm$$

n^+ poly on $n \Rightarrow \phi_{ms} = -0.32\ V$

We have

$$\left|Q'_{SD}(\max)\right| = \left(1.6x10^{-19}\right)\left(10^{17}\right)\left(1.026x10^{-5}\right)$$

or

$$\left|Q'_{SD}(\max)\right| = 1.64x10^{-7}\ C/cm^2$$

Now

$$V_{TP} = \left[-1.64x10^{-7} - \left(1.6x10^{-19}\right)\left(5x10^{10}\right)\right]$$

$$\times\frac{\left(80x10^{-8}\right)}{(3.9)\left(8.85x10^{-14}\right)} - 0.32 - 2(0.407)$$

or

$V_{TP} = -1.53\ V$, Enhancement PMOS

(b)
For $V_T = 0$, shift threshold voltage in positive direction, so implant acceptor ions

$$\Delta V_T = \frac{eD_I}{C_{ox}} \Rightarrow D_I = \frac{(\Delta V_T)C_{ox}}{e}$$

so

$$D_I = \frac{(1.53)(3.9)\left(8.85x10^{-14}\right)}{\left(80x10^{-8}\right)\left(1.6x10^{-19}\right)}$$

or

$$D_I = 4.13x10^{12}\ cm^{-2}$$

12.38

Shift in negative direction means implanting donor ions. We have

$$\Delta V_T = \frac{eD_I}{C_{ox}}$$

where

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)\left(8.85x10^{-14}\right)}{400x10^{-8}}$$

or

$$C_{ox} = 8.63x10^{-8}\ F/cm^2$$

Now

$$D_I = \frac{C_{ox}(\Delta V_T)}{e} = \frac{(8.63x10^{-8})(1.4)}{1.6x10^{-19}}$$

or

$$\underline{D_I = 7.55x10^{11}\ cm^{-2}}$$

12.39

The areal density of generated holes is

$$= (8x10^{12})(10^5)(750x10^{-8}) = 6x10^{12}\ cm^{-2}$$

The equivalent surface charge trapped is

$$= (0.10)(6x10^{12}) = 6x10^{11}\ cm^{-2}$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{(6x10^{11})(1.6x10^{-19})}{(3.9)(8.85x10^{-14})}(750x10^{-8})$$

or

$$\underline{\Delta V_T = -2.09\ V}$$

12.40

The areal density of generated holes is $6x10^{12}\ cm^{-2}$. Now

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{750x10^{-8}}$$

or

$$C_{ox} = 4.6x10^{-8}\ F/cm^2$$

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{(6x10^{12})(x)(1.6x10^{-19})}{4.6x10^{-8}}$$

For $\Delta V_T = -0.50\ V$

Where the parameter x is the maximum fraction of holes that can be trapped. Then we find

$$\underline{x = 0.024 \Rightarrow 2.4\%}$$

12.41

We have the areal density of generated holes as $= (g)(\gamma)(t_{ox})$ where g is the generation rate and γ is the dose. The equivalent charge trapped is $= xg\gamma t_{ox}$.

Then

$$\Delta V_T = -\frac{Q'_{SS}}{C_{ox}} = -\frac{exg\gamma t_{ox}}{(\in_{ox}/t_{ox})} = -exg\gamma(t_{ox})^2$$

so that

$$\underline{\Delta V_T \propto -(t_{ox})^2}$$