Chapter 13

Problem Solutions

13.1
Sketch

13.2
Sketch

13.3
p-channel JFET – Silicon
(a)

$$V_{PO} = \frac{ea^2 N_a}{2\in} = \frac{(1.6x10^{-19})(0.5x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.79\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.79 - 0.884$$

or

$$\underline{V_P = 4.91\ V}$$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i)
For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(0.884 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - \left[(4.31x10^{-10})(1.884 - V_{DS})\right]^{1/2}$$

or

$$\underline{a - h = 0.215\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.0653\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.045\ \mu m}$$

which implies no undepleted region.

13.4
p-channel JFET – GaAs
(a)

$$V_{PO} = \frac{2a^2 N_a}{2\in} = \frac{2(0.5x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 5.18\ V}$$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.35\ V$$

so

$$V_P = V_{PO} - V_{bi} = 5.18 - 1.35$$

or

$$\underline{V_P = 3.83\ V}$$

(b)

$$a - h = a - \left[\frac{2\in(V_{bi} - V_{DS} + V_{GS})}{eN_a}\right]^{1/2}$$

(i) For $V_{GS} = 1\ V$, $V_{DS} = 0$
Then

$$a - h = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(1.35 + 1 - V_{DS})}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$$a - h = 0.5x10^{-4} - \left[(4.83x10^{-10})(2.35 - V_{DS})\right]^{1/2}$$

which yields

$$\underline{a - h = 0.163\ \mu m}$$

(ii) For $V_{GS} = 1\ V$, $V_{DS} = -2.5\ V$

$$\underline{a - h = 0.016\ \mu m}$$

(iii) For $V_{GS} = 1\ V$, $V_{DS} = -5\ V$

$$\underline{a - h = -0.096\ \mu m}$$

which implies no undepleted region.

13.5

(a) $V_{PO} = \dfrac{ea^2 N_d}{2 \in}$

$= \dfrac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(11.7)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 15.5\ V}$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(11.7)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

or

$9x10^{-10} = 1.618x10^{-10}(V_{bi} - V_{GS})$

which yields

$V_{bi} - V_{GS} = 5.56\ V$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.896\ V$

Then

$V_{GS} = 0.897 - 5.56 \Rightarrow \underline{V_{GS} = -4.66\ V}$

13.6

For GaAs:

(a)

$V_{PO} = \dfrac{ea^2 N_d}{2 \in}$

$= \dfrac{(1.6x10^{-19})(0.5x10^{-4})^2(8x10^{16})}{2(13.1)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 13.8\ V}$

(b)

$$a - h = a - \left[\frac{2 \in (V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

$$0.2x10^{-4} = 0.5x10^{-4} - \left[\frac{2(13.1)(8.85x10^{-14})(V_{bi} - V_{GS})}{(1.6x10^{-19})(8x10^{16})}\right]^{1/2}$$

which can be written as

$9x10^{-10} = 1.811x10^{-10}(V_{bi} - V_{GS})$

or

$V_{bi} - V_{GS} = 4.97\ V$

Now

$$V_{bi} = (0.0259)\ln\left[\frac{(3x10^{18})(8x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$V_{bi} = 1.36\ V$

Then

$V_{GS} = V_{bi} - 4.97 = 1.36 - 4.97$

or

$\underline{V_{GS} = -3.61\ V}$

13.7

(a) $V_{PO} = \dfrac{ea^2 N_a}{2 \in}$

$= \dfrac{(1.6x10^{-19})(0.3x10^{-4})^2(3x10^{16})}{2(13.1)(8.85x10^{-14})}$

or

$\underline{V_{PO} = 1.863\ V}$

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(3x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$V_{bi} = 1.352\ V$

Then

$V_P = V_{PO} - V_{bi} = 1.863 - 1.352$

or

$\underline{V_P = 0.511\ V}$

(b) (i)

$$a - h = a - \left[\frac{2 \in (V_{bi} + V_{GS})}{eN_a}\right]^{1/2}$$

or

$$a - h = (0.3x10^{-4}) - \left[\frac{2(13.1)(8.85x10^{-14})(1.352)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

which yields

$$a-h=4.45x10^{-6}\ cm$$

(ii)

$$a-h=\left(0.3x10^{-4}\right)-\left[\frac{2(13.1)\left(8.85x10^{-14}\right)(1.351+1)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

which yields

$$a-h=-3.7x10^{-6}\ cm$$

which implies no undepleted region.

13.8

(a) n-channel JFET – Silicon

$$V_{PO}=\frac{ea^2N_d}{2\in}=\frac{\left(1.6x10^{-19}\right)\left(0.35x10^{-4}\right)^2\left(4x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO}=3.79\ V$$

Now

$$V_{bi}=(0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi}=0.892\ V$$

so that

$$V_P=V_{bi}-V_{PO}=0.892-3.79$$

or

$$V_P=-2.90\ V$$

(b)

$$a-h=a-\left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_d}\right]^{1/2}$$

We have

$$a-h=0.35x10^{-4}-\left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(0.892+V_{DS}-V_{GS}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$a-h=0.35x10^{-4}-\left[\left(3.24x10^{-10}\right)\left(0.892+V_{DS}-V_{GS}\right)\right]^{1/2}$$

(i) For $V_{GS}=0$, $V_{DS}=1\ V$,

$$a-h=0.102\ \mu m$$

(ii) For $V_{GS}=-1\ V$, $V_{DS}=1\ V$,

$$a-h=0.044\ \mu m$$

(iii) For $V_{GS}=-1\ V$, $V_{DS}=2\ V$,

$$a-h=-0.0051\ \mu m$$

which implies no undepleted region

13.9

$$V_{bi}=(0.0259)\ln\left(\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.8x10^{6}\right)^2}\right)$$

or

$$V_{bi}=1.359\ V$$

$$a-h=a-\left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_D}\right]^{1/2}$$

or

$$a-h=0.35x10^{-4}-\left[\frac{2(13.1)\left(8.85x10^{-14}\right)\left(1.359+V_{DS}-V_{GS}\right)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

We want $a-h=0.05x10^{-4}\ cm$,

Then

$$0.05x10^{-4}=0.35x10^{-4}-\left[\left(3.623x10^{-10}\right)\left(1.359+V_{DS}-V_{GS}\right)\right]^{1/2}$$

(a)

For $V_{DS}=0$, we find

$$V_{GS}=-1.125\ V$$

(b)

For $V_{DS}=1\ V$, we find

$$V_{GS}=-0.125\ V$$

13.10

(a)

$$I_{P1}=\frac{\mu_n\left(eN_d\right)^2Wa^3}{6\in L}$$

$$=\frac{(1000)\left[\left(1.6x10^{-19}\right)\left(10^{16}\right)\right]^2}{6(11.7)\left(8.85x10^{-14}\right)}\times\frac{\left(400x10^{-4}\right)\left(0.5x10^{-4}\right)^3}{\left(20x10^{-4}\right)}$$

or

$$I_{P1}=1.03\ mA$$

(b)

$$V_{PO}=\frac{ea^2N_d}{2\in}$$

$$= \left[\frac{(1.6x10^{-19})(0.5x10^{-4})^2(10^{16})}{2(11.7)(8.85x10^{-14})}\right]$$

or

$$\underline{V_{PO} = 1.93\ V}$$

Also

$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(10^{16})}{(1.5x10^{10})^2}\right]$$

or

$V_{bi} = 0.874\ V$

Now

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= 1.93 - 0.874 + V_{GS}$$

or

$V_{DS}(sat) = 1.06 + V_{GS}$

We have

$V_P = V_{bi} - V_{PO} = 0.874 - 1.93$

or

$$\underline{V_P = -1.06\ V}$$

Then

(i) $V_{GS} = 0 \Rightarrow$ $\underline{V_{DS}(sat) = 1.06\ V}$

(ii) $V_{GS} = \frac{1}{4}V_P = -0.265\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.795\ V}$

(iii) $V_{GS} = \frac{1}{2}V_P = -0.53\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.53\ V}$

(iv) $V_{GS} = \frac{3}{4}V_P = -0.795\ V \Rightarrow$
$\underline{V_{DS}(sat) = 0.265\ V}$

(c)

$I_{D1}(sat)$

$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

$$= 1.03\left[1 - 3\left(\frac{0.874 - V_{GS}}{1.93}\right) \times \left(1 - \frac{2}{3}\sqrt{\frac{0.874 - V_{GS}}{1.93}}\right)\right]$$

(i) For $V_{GS} = 0 \Rightarrow$ $\underline{I_{D1}(sat) = 0.258\ mA}$

(ii) For $V_{GS} = -0.265\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.140\ mA}$

(iii) For $V_{GS} = -0.53\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.061\ mA}$

(iv) For $V_{GS} = -0.795\ V \Rightarrow$
$\underline{I_{D1}(sat) = 0.0145\ mA}$

13.11

$$g_d = G_{O1}\left[1 - \left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)^{1/2}\right]$$

where

$$G_{O1} = \frac{3I_{P1}}{V_{PO}} = \frac{3(1.03x10^{-3})}{1.93} = 1.60x10^{-3}$$

or

$G_{O1} = 1.60\ mS$

Then

V_{GS}	$[(V_{bi} - V_{GS})/V_{PO}]$	$g_d(mS)$
0	0.453	0.523
-0.265	0.590	0.371
-0.53	0.727	0.236
-0.795	0.945	0.112
-1.06	1.0	0

13.12

n-channel JFET – GaAs

(a)

$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$

$$= \frac{(1.6x10^{-19})(8000)(2x10^{16})(30x10^{-4})(0.35x10^{-4})}{10x10^{-4}}$$

or

$$\underline{G_{O1} = 2.69x10^{-3}\ S}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.35x10^{-4})^2(2x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 1.69\ V}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{(5x10^{18})(2x10^{16})}{(1.8x10^{6})^2}\right]$$

or

$$V_{bi} = 1.34\ V$$

Then

$$V_P = V_{bi} - V_{PO} = 1.34 - 1.69$$

or

$$\underline{V_P = -0.35\ V}$$

We then obtain

$$V_{DS}(sat) = 1.69 - (1.34 - V_{GS}) = 0.35 + V_{GS}$$

For $V_{GS} = 0 \Rightarrow$ $\underline{V_{DS}(sat) = 0.35\ V}$

For $V_{GS} = \frac{1}{2}V_P = -0.175\ V \Rightarrow$

$$\underline{V_{DS}(sat) = 0.175\ V}$$

(c)

$$I_{D1}(sat)$$

$$= I_{P1}\left[1 - 3\left(\frac{V_{bi} - V_{GS}}{V_{PO}}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)\right]$$

where

$$I_{P1} = \frac{\mu_n (eN_d)^2 Wa^3}{6 \in L}$$

$$= \frac{(8000)\left[(1.6x10^{-19})(2x10^{16})\right]^2}{6(13.1)(8.85x10^{-14})}$$

$$\times \frac{(30x10^{-4})(0.35x10^{-4})^3}{(10x10^{-4})}$$

or

$$\underline{I_{P1} = 1.51\ mA}$$

Then

$$I_{D1}(sat) = 1.51\left[1 - 3\left(\frac{1.34 - V_{GS}}{1.69}\right)\right.$$

$$\left.\times\left(1 - \frac{2}{3}\sqrt{\frac{1.34 - V_{GS}}{1.69}}\right)\right](mA)$$

For

$V_{GS} = 0 \Rightarrow$ $\underline{I_{D1}(sat) = 0.0504\ mA}$

and for

$V_{GS} = -0.175\ V \Rightarrow$ $\underline{I_{D1}(sat) = 0.0123\ mA}$

13.13

$$g_{mS} = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi} - V_{GS}}{V_{PO}}}\right)$$

We have

$$I_{P1} = 1.03\ mA,\ V_{PO} = 1.93\ V,\ V_{bi} = 0.874\ V$$

The maximum transconductance occurs when $V_{GS} = 0$

Then

$$g_{mS}(\max) = \frac{3(1.03)}{1.93}\left(1 - \sqrt{\frac{0.874}{1.93}}\right)$$

or

$$g_{mS} = 0.524\ mS$$

For $W = 400\ \mu m$,

We have

$$g_{mS}(\max) = \frac{0.524\ mS}{400x10^{-4}\ cm}$$

or

$$\underline{g_{mS} = 13.1\ mS/cm = 1.31\ mS/mm}$$

13.14

The maximum transconductance occurs for $V_{GS} = 0$, so we have

(a)

$$g_{mS}(\max) = \frac{3I_{P1}}{V_{PO}}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

$$= G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

We found

$$G_{O1} = 2.69\ mS,\ V_{bi} = 1.34\ V,\ V_{PO} = 1.69\ V$$

Then

$$g_{mS}(\max) = (2.69)\left(1 - \sqrt{\frac{1.34}{1.69}}\right)$$

or

$$\underline{g_{mS}(\max) = 0.295\ mS}$$

This is for a channel length of $L = 10\ \mu m$.

(b)

If the channel length is reduced to $L = 2\ \mu m$, then

$$g_{mS}(\max) = (0.295)\left(\frac{10}{2}\right) \Rightarrow$$

$$\underline{g_{mS}(\max) = 1.48\ mS}$$

13.15
n-channel MESFET – GaAs
(a)

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.5x10^{-4})^2(1.5x10^{16})}{2(13.1)(8.85x10^{-14})}$$

or

$$V_{PO} = 2.59\ V$$

Now

$$V_{bi} = \phi_{Bn} - \phi_n$$

where

$$\phi_n = V_t \ln\left(\frac{N_C}{N_d}\right) = (0.0259)\ln\left(\frac{4.7x10^{17}}{1.5x10^{16}}\right)$$

or

$$\phi_n = 0.0892\ V$$

so that

$$V_{bi} = 0.90 - 0.0892 = 0.811\ V$$

Then

$$V_T = V_{bi} - V_{PO} = 0.811 - 2.59$$

or

$$\underline{V_T = -1.78\ V}$$

(b)
If $V_T < 0$ for an n-channel device, the device is a depletion mode MESFET.

13.16
n-channel MESFET – GaAs
(a)
We want $V_T = +0.10\ V$
Then

$$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$$

so

$$V_T = 0.10 = 0.89 - V_t \ln\left(\frac{N_C}{N_d}\right) - \frac{ea^2 N_d}{2\in}$$

which can be written as

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right)$$

$$+ \frac{(1.6x10^{-19})(0.35x10^{-4})^2 N_d}{2(13.1)(8.85x10^{-14})} = 0.89 - 0.10$$

or

$$(0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) + (8.45x10^{17})N_d = 0.79$$

By trial and error

$$\underline{N_d = 8.1x10^{15}\ cm^{-3}}$$

(b)
At $T = 400K$,

$$N_C(400) = N_C(300)\cdot\left(\frac{400}{300}\right)^{3/2}$$

$$= (4.7x10^{17})(1.54)$$

or

$$N_C(400) = 7.24x10^{17}\ cm^{-3}$$

Also

$$V_t = (0.0259)\left(\frac{400}{300}\right) = 0.03453$$

Then

$$V_T = 0.89 - (0.03453)\ln\left(\frac{7.24x10^{17}}{8.1x10^{15}}\right)$$

$$-(8.45x10^{-17})(8.1x10^{15})$$

which becomes

$$\underline{V_T = +0.051\ V}$$

13.17
We have

$$a - h = a - \left[\frac{2\in(V_{bi} + V_{DS} - V_{GS})}{eN_d}\right]^{1/2}$$

where

$$V_{bi} = \phi_{Bn} - \phi_n$$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{5x10^{16}}\right) = 0.058\ V$$

Then

$$V_{bi} = 0.80 - 0.058 = 0.742\ V$$

For $V_{GS} = 0.5\ V$,

$$a - h = (0.8x10^{-4})$$

$$-\left[\frac{2(13.1)(8.85x10^{-14})(0.742 + V_{DS} - 0.5)}{(1.6x10^{-19})(5x10^{16})}\right]^{1/2}$$

or

$$a - h = (0.80x10^{-4})$$

$$-\left[(2.898x10^{-10})(0.242 + V_{DS})\right]^{1/2}$$

Then

$V_{DS}(V)$	$a-h\ (\mu m)$
0	0.716
1	0.610
2	0.545
5	0.410

13.18

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want

$V_T = 0 \Rightarrow \phi_n + V_{PO} = \phi_{Bn}$

Device 1: $N_d = 3x10^{16}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{16}}\right) = 0.0713\ V$$

so that

$V_{PO} = 0.89 - 0.0713 = 0.8187\ V$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8187)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$

or

$\underline{a = 0.199\ \mu m}$

Device 2: $N_d = 3x10^{17}\ cm^{-3}$

Then

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{3x10^{17}}\right) = 0.0116\ V$$

so that

$V_{PO} = 0.89 - 0.0116 = 0.8784\ V$

Now

$$a = \left[\frac{2\in V_{PO}}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(13.1)(8.85x10^{-14})(0.8784)}{(1.6x10^{-19})(3x10^{17})}\right]^{1/2}$$

or

$\underline{a = 0.0651\ \mu m}$

13.19

$V_T = V_{bi} - V_{PO} = \phi_{Bn} - \phi_n - V_{PO}$

We want $V_T = 0.5\ V$, so

$0.5 = 0.85 - \phi_n - V_{PO}$

Now

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right)$$

and

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{(1.6x10^{-19})(0.25x10^{-4})^2 N_d}{2(13.1)(8.85x10^{-14})}$$

or

$V_{PO} = (4.31x10^{-17})N_d$

Then

$$0.5 = 0.85 - (0.0259)\ln\left(\frac{4.7x10^{17}}{N_d}\right) - (4.31x10^{-17})N_d$$

By trial and error, we find

$\underline{N_d = 5.45x10^{15}\ cm^{-3}}$

13.20

n-channel MESFET – silicon

(a) For a gold contact, $\phi_{Bn} = 0.82\ V$.

We find

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{10^{16}}\right) = 0.206\ V$$

and

$V_{bi} = \phi_{Bn} - \phi_n = 0.82 - 0.206 = 0.614\ V$

With $V_{DS} = 0$, $V_{GS} = 0.35\ V$

We find

$$a - h = 0.075x10^{-4} = a - \left[\frac{2\in(V_{bi} - V_{GS})}{eN_d}\right]^{1/2}$$

so that

$$a = 0.075x10^{-4} + \left[\frac{2(11.7)(8.85x10^{-14})(0.614-0.35)}{(1.6x10^{-19})(10^{16})}\right]^{1/2}$$

or

$a = 0.26\ \mu m$

Now

$$V_T = V_{bi} - V_{PO} = 0.614 - \frac{ea^2 N_d}{2\in}$$

or

$$V_T = 0.614 - \frac{(1.6x10^{-19})(0.26x10^{-4})^2(10^{16})}{2(11.7)(8.85x10^{-14})}$$

We obtain

$$\underline{V_T = 0.092\ V}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= (V_{bi} - V_T) - (V_{bi} - V_{GS}) = V_{GS} - V_T$$

Now

$$V_{DS}(sat) = 0.35 - 0.092$$

or

$$\underline{V_{DS}(sat) = 0.258\ V}$$

13.21

(a) n-channel MESFET - silicon

$$V_{bi} = \phi_{Bn} - \phi_n$$

and

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{2x10^{16}}\right) = 0.188\ V$$

so

$$V_{bi} = 0.80 - 0.188 \Rightarrow \underline{V_{bi} = 0.612\ V}$$

Now

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$
$$= \frac{(1.6x10^{-19})(0.4x10^{-4})^2(2x10^{16})}{2(11.7)(8.85x10^{-14})}$$

or

$$\underline{V_{PO} = 2.47\ V}$$

We find

$$V_T = V_{bi} - V_{PO} = 0.612 - 2.47$$

or

$$\underline{V_T = -1.86\ V}$$

and

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= 2.47 - (0.612 - (-1))$$

or

$$\underline{V_{DS}(sat) = 0.858\ V}$$

(b)

For $V_{PO} = 4.5\ V$, additional donor atoms must be added.

We have

$$V_{PO} = \frac{ea^2 N_d}{2\in} \Rightarrow N_d = \frac{2\in V_{PO}}{ea^2}$$

so that

$$N_d = \frac{2(11.7)(8.85x10^{-14})(4.5)}{(1.6x10^{-19})(0.4x10^{-4})^2}$$

or

$$\underline{N_d = 3.64x10^{16}\ cm^{-3}}$$

which means that

$$\Delta N_d = 3.64x10^{16} - 2x10^{16}$$

or

$$\underline{\Delta N_d = 1.64x10^{16}\ cm^{-3}}$$

Donors must be added

Then

$$\phi_n = (0.0259)\ln\left(\frac{2.8x10^{19}}{3.64x10^{16}}\right) = 0.172\ V$$

so that

$$V_{bi} = 0.80 - 0.172 = 0.628\ V$$

We find

$$V_T = V_{bi} - V_{PO} = 0.628 - 4.5$$

or

$$\underline{V_T = -3.87\ V}$$

Also

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS})$$
$$= 4.5 - (0.628 - (-1))$$

or

$$\underline{V_{DS}(sat) = 2.87\ V}$$

13.22

(a) $$k_n = \frac{\mu_n \in W}{2aL}$$
$$= \frac{(7800)(13.1)(8.85x10^{-14})(20x10^{-4})}{2(0.30x10^{-4})(1.2x10^{-4})}$$

or

$$\underline{k_n = 2.51\ mA/V^2}$$

(b)

$$V_{DS}(sat) = V_{PO} - (V_{bi} - V_{GS}) = V_{GS} - V_T$$

So for $V_{GS} = 1.5V_T \Rightarrow V_{DS}(sat) = (0.5)(0.12)$

Or

$$\underline{V_{DS}(sat) = 0.06\ V}$$

and for $V_{GS} = 2V_T \Rightarrow V_{DS}(sat) = (1)(0.12)$

or

$$\underline{V_{DS}(sat) = 0.12\ V}$$

(c)

$$I_{D1}(sat) = k_n(V_{GS} - V_T)^2$$

For $V_{GS} = 1.5V_T \Rightarrow I_{D1}(sat) = (2.51)(0.06)^2$
Or
$$I_{D1}(sat) = 9.04\ \mu A$$
and for $V_{GS} = 2V_T \Rightarrow I_{D1}(sat) = (2.51)(0.12)^2$
or
$$I_{D1}(sat) = 36.1\ \mu A$$

13.23
(a) We have
$$g_m = 2k_n(V_{GS} - V_T)$$
so that
$$1.75x10^{-3} = 2k_n(0.50 - 0.25)$$
which gives
$$k_n = 3.5x10^{-3}\ A/V^2 = \frac{\mu_n \in W}{2aL}$$
We obtain
$$W = \frac{(3.5x10^{-3})(2)(0.35x10^{-4})(10^{-4})}{(8000)(13.1)(8.85x10^{-14})}$$
or
$$W = 26.4\ \mu m$$
(b)
$$I_{D1}(sat) = k_n(V_{GS} - V_T)^2$$
For $V_{GS} = 0.4\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.4 - 0.25)^2$$
or
$$I_{D1}(sat) = 78.8\ \mu A$$
For $V_{GS} = 0.65\ V$,
$$I_{D1}(sat) = (3.5x10^{-3})(0.65 - 0.25)^2$$
or
$$I_{D1}(sat) = 0.56\ mA$$

13.24
Computer plot

13.25
Computer plot

13.26

We have $L' = L - \frac{1}{2}\Delta L$

Or
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
We have
$$\Delta L = \left[\frac{2 \in (V_{DS} - V_{DS}(sat))}{eN_d}\right]^{1/2}$$
and
For $V_{GS} = 0$, $V_{DS}(sat) = V_{PO} - V_{bi}$
We find
$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$
$$= \frac{(1.6x10^{-19})(0.4x10^{-4})^2(3x10^{16})}{2(11.7)(8.85x10^{-14})}$$
or
$$V_{PO} = 3.71\ V$$
and
$$V_{bi} = (0.0259)\ln\left[\frac{(10^{19})(3x10^{16})}{(1.5x10^{10})^2}\right]$$
or
$$V_{bi} = 0.902\ V$$
so that
$$V_{DS}(sat) = 3.71 - 0.902 = 2.81\ V$$
Then
$$\Delta L = \left[\frac{2(11.7)(8.85x10^{-14})(5 - 2.81)}{(1.6x10^{-19})(3x10^{16})}\right]^{1/2}$$
or
$$\Delta L = 0.307\ \mu m$$
Now
$$\frac{L'}{L} = 0.90 = 1 - \frac{1}{2}\cdot\frac{\Delta L}{L}$$
or
$$\frac{1}{2}\cdot\frac{\Delta L}{L} = 1 - 0.9 = 0.10$$
so
$$L = \frac{\Delta L}{2(0.10)} = \frac{0.307x10^{-4}}{2(0.10)}$$
or
$$L = 1.54\ \mu m$$

13.27

We have that $I'_{D1} = I_{D1}\left(\frac{L}{L-(1/2)\Delta L}\right)$

Assuming that we are in the saturation region, then $I'_{D1} = I'_{D1}(sat)$ and $I_{D1} = I_{D1}(sat)$. We can write

$$I'_{D1}(sat) = I_{D1}(sat)\cdot\frac{1}{1-\frac{1}{2}\cdot\frac{\Delta L}{L}}$$

If $\Delta L << L$, then

$$I'_{D1}(sat) = I_{D1}(sat)\left[1+\frac{1}{2}\cdot\frac{\Delta L}{L}\right]$$

We have that

$$\Delta L = \left[\frac{2\in\left(V_{DS}-V_{DS}(sat)\right)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2\in V_{DS}}{eN_d}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

which can be written as

$$\Delta L = V_{DS}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

If we write

$$I'_{D1}(sat) = I_{D1}(sat)(1+\lambda V_{DS})$$

then by comparing equations, we have

$$\lambda = \frac{1}{2L}\left[\frac{2\in}{eN_dV_{DS}}\left(1-\frac{V_{DS}(sat)}{V_{DS}}\right)\right]^{1/2}$$

The parameter is not independent of V_{DS}. Define

$x = \frac{V_{DS}}{V_{DS}(sat)}$ and consider the function

$f = \frac{1}{x}\left(1-\frac{1}{x}\right)$ which is directly proportional to λ. We find that

x	$f(x)$
1..5	0.222
1.75	0.245
2.0	0.250
2.25	0.247
2.50	0.240
2.75	0.231
3.0	0.222

So that λ is nearly a constant.

13.28

(a) Saturation occurs when $\mathrm{E} = 1x10^4\ V/cm$

As a first approximation, let

$$\mathrm{E} = \frac{V_{DS}}{L}$$

Then

$$V_{DS} = \mathrm{E}\cdot L = \left(1x10^4\right)\left(2x10^{-4}\right)$$

or

$$\underline{V_{DS} = 2\ V}$$

(b)

We have that

$$h_2 = h_{sat} = \left[\frac{2\in\left(V_{bi}+V_{DS}-V_{GS}\right)}{eN_d}\right]^{1/2}$$

and

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(4x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.892\ V$$

For $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.892+2)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$\underline{h_{sat} = 0.306\ \mu m}$$

(c)

We then find

$$I_{D1}(sat) = eN_dv_{sat}\left(a-h_{sat}\right)W$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(10^7\right)(0.50-0.306)$$

$$\times\left(10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$\underline{I_{D1}(sat) = 3.72\ mA}$$

(d)

For $V_{GS} = 0$, we have

$$I_{D1}(sat) = I_{P1}\left[1-3\left(\frac{V_{bi}}{V_{PO}}\right)\left(1-\frac{2}{3}\sqrt{\frac{V_{bi}}{V_{PO}}}\right)\right]$$

Now

$$I_{P1} = \frac{\mu_n\left(eN_d\right)^2Wa^3}{6\in L}$$

$$= \frac{(1000)\left[\left(1.6x10^{-19}\right)\left(4x10^{16}\right)\right]^2}{6(11.7)\left(8.85x10^{-14}\right)}$$

$$\times \frac{\left(30x10^{-4}\right)\left(0.5x10^{-4}\right)^3}{\left(2x10^{-4}\right)}$$

or

$$I_{P1} = 12.4\ mA$$

Also

$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.5x10^{-4}\right)^2\left(4x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$$V_{PO} = 7.73\ V$$

Then

$$I_{D1}(sat) = 12.4\left[1 - 3\left(\frac{0.892}{7.73}\right)\left(1 - \frac{2}{3}\sqrt{\frac{0.892}{7.73}}\right)\right]$$

or

$$\underline{I_{D1}(sat) = 9.08\ mA}$$

13.29

(a) If $L = 1\ \mu m$, then saturation will occur when

$$V_{DS} = \mathrm{E} \cdot L = \left(10^4\right)\left(1x10^{-4}\right) = 1\ V$$

We find

$$h_2 = h_{sat} = \left[\frac{2 \in \left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

We have $V_{bi} = 0.892\ V$ and for $V_{GS} = 0$, we obtain

$$h_{sat} = \left[\frac{2(11.7)\left(8.85x10^{-14}\right)(0.892 + 1)}{\left(1.6x10^{-19}\right)\left(4x10^{16}\right)}\right]^{1/2}$$

or

$$h_{sat} = 0.247\ \mu m$$

Then

$$I_{D1}(sat) = eN_d v_{sat}\left(a - h_{sat}\right)W$$

$$= \left(1.6x10^{-19}\right)\left(4x10^{16}\right)\left(10^7\right)(0.50 - 0.247)$$

$$\times\left(10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$\underline{I_{D1}(sat) = 4.86\ mA}$$

If velocity saturation did not occur, then from the previous problem, we would have

$$I_{D1}(sat) = 9.08\left(\frac{2}{1}\right) \Rightarrow \underline{I_{D1}(sat) = 18.2\ mA}$$

(b)
If velocity saturation occurs, then the relation $I_{D1}(sat) \propto (1/L)$ does not apply.

13.30

(a)

$$v = \mu_n \mathrm{E} = (8000)\left(5x10^3\right) = 4x10^7\ cm/s$$

Then

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{4x10^7} \Rightarrow$$

or

$$\underline{t_d = 5\ ps}$$

(b)
Assume $v_{sat} = 10^7\ cm/s$

Then

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow$$

$$\underline{t_d = 20\ ps}$$

13.31

(a) $v = \mu_n \mathrm{E} = (1000)\left(10^4\right) = 10^7\ cm/s$

$$t_d = \frac{L}{v} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

(b)
For $v_{sat} = 10^7\ cm/s$,

$$t_d = \frac{L}{v_{sat}} = \frac{2x10^{-4}}{10^7} \Rightarrow \underline{t_d = 20\ ps}$$

13.32

The reverse-bias current is dominated by the generation current. We have

$$V_P = V_{bi} - V_{PO}$$

We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(5x10^{18}\right)\left(3x10^{16}\right)}{\left(1.5x10^{10}\right)^2}\right]$$

or

$$V_{bi} = 0.884\ V$$

and

$$V_{PO} = \frac{ea^2 N_d}{2 \in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(3x10^{16}\right)}{2(11.7)\left(8.85x10^{-14}\right)}$$

or

$V_{PO} = 2.09\ V$

Then

$V_P = 0.884 - 2.09 = -1.21 = V_{GS}$

Now

$$x_n = \left[\frac{2\in\left(V_{bi} + V_{DS} - V_{GS}\right)}{eN_d}\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85x10^{-14}\right)\left(0.884 + V_{DS} - (-1.21)\right)}{\left(1.6x10^{-19}\right)\left(3x10^{16}\right)}\right]^{1/2}$$

or

$$x_n = \left[\left(4.31x10^{-10}\right)\left(2.09 + V_{DS}\right)\right]^{1/2}$$

(a)

For $V_{DS} = 0 \Rightarrow$, $x_n = 0.30\ \mu m$

(b)

For $V_{DS} = 1\ V \Rightarrow$, $x_n = 0.365\ \mu m$

(c)

For $V_{DS} = 5\ V \Rightarrow x_n = 0.553\ \mu m$

The depletion region volume is

$$Vol = (a)\left(\frac{L}{2}\right)(W) + \left(x_n\right)(2a)(W)$$

$$= \left(0.3x10^{-4}\right)\left(\frac{2.4x10^{-4}}{2}\right)\left(30x10^{-4}\right) + \left(x_n\right)\left(0.6x10^{-4}\right)\left(30x10^{-4}\right)$$

or

$$Vol = 10.8x10^{-12} + x_n\left(18x10^{-8}\right)$$

(a)

For $V_{DS} = 0 \Rightarrow\ Vol = 1.62x10^{-11}\ cm^3$

(b)

For $V_{DS} = 1\ V \Rightarrow\ Vol = 1.74x10^{-11}\ cm^3$

(c)

For $V_{DS} = 5\ V \Rightarrow\ Vol = 2.08x10^{-11}\ cm^3$

The generation current is

$$I_{DG} = e\left(\frac{n_i}{2\tau_O}\right)\cdot Vol = \frac{\left(1.6x10^{-19}\right)\left(1.5x10^{10}\right)}{2\left(5x10^{-8}\right)}\cdot Vol$$

or

$$I_{DG} = \left(2.4x10^{-2}\right)\cdot Vol$$

(a)

For $V_{DS} = 0 \Rightarrow\ \underline{I_{DG} = 0.39\ pA}$

(b)

For $V_{DS} = 1\ V \Rightarrow\ \underline{I_{DG} = 0.42\ pA}$

(c)

For $V_{DS} = 5\ V \Rightarrow\ \underline{I_{DG} = 0.50\ pA}$

13.33

(a) The ideal transconductance for $V_{GS} = 0$ is

$$g_{mS} = G_{O1}\left(1 - \sqrt{\frac{V_{bi}}{V_{PO}}}\right)$$

where

$$G_{O1} = \frac{e\mu_n N_d Wa}{L}$$

$$= \frac{\left(1.6x10^{-19}\right)(4500)\left(7x10^{16}\right)}{1.5x10^{-4}} \times \left(5x10^{-4}\right)\left(0.3x10^{-4}\right)$$

or

$G_{O1} = 5.04\ mS$

We find

$$V_{PO} = \frac{ea^2 N_d}{2\in}$$

$$= \frac{\left(1.6x10^{-19}\right)\left(0.3x10^{-4}\right)^2\left(7x10^{16}\right)}{2(13.1)\left(8.85x10^{-14}\right)}$$

or

$V_{PO} = 4.35\ V$

We have

$$\phi_n = (0.0259)\ln\left(\frac{4.7x10^{17}}{7x10^{16}}\right) = 0.049\ V$$

so that

$V_{bi} = \phi_{Bn} - \phi_n = 0.89 - 0.049 = 0.841\ V$

Then

$$g_{mS} = 5.04\left(1 - \sqrt{\frac{0.841}{4.35}}\right)$$

or

$\underline{g_{mS} = 2.82\ mS}$

(b)

With a source resistance

$$g'_m = \frac{g_m}{1 + g_m r_s} \Rightarrow \frac{g'_m}{g_m} = \frac{1}{1 + g_m r_s}$$

For

$$\frac{g'_m}{g_m} = 0.80 = \frac{1}{1 + (2.82)r_s}$$

which yields

$$\underline{r_s = 88.7\ \Omega}$$

(c)

$$r_s = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{(e\mu_n n)(0.3x10^{-4})(5x10^{-4})}$$

so

$$L = (88.7)(1.6x10^{-19})(4500)(7x10^{16}) \times (0.3x10^{-4})(5x10^{-4})$$

or

$$\underline{L = 0.67\ \mu m}$$

13.34

$$f_T = \frac{g_m}{2\pi C_G}$$

where

$$C_G = \frac{\in WL}{a} = \frac{(13.1)(8.85x10^{-14})(5x10^{-4})(1.5x10^{-4})}{0.3x10^{-4}}$$

or

$$C_G = 2.9x10^{-15}\ F$$

We must use g'_m, so we obtain

$$f_T = \frac{(2.82x10^{-3})(0.80)}{2\pi(2.9x10^{-15})} = 124\ GHz$$

We have

$$f_T = \frac{1}{2\pi\tau_C} \Rightarrow \tau_C = \frac{1}{2\pi f_T} = \frac{1}{2\pi(124x10^9)}$$

or

$$\tau_C = 1.28x10^{-12}\ s$$

The channel transit time is

$$t_t = \frac{1.5x10^{-4}}{10^7} = 1.5x10^{-11}\ s$$

The total time constant is

$$\tau = 1.5x10^{-11} + 1.28x10^{-12} = 1.63x10^{-11}\ s$$

so that

$$f_T = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.63x10^{-11})}$$

or

$$\underline{f_T = 9.76\ GHz}$$

13.35

(a) For a constant mobility

$$f_T = \frac{e\mu_n N_d a^2}{2\pi \in L^2} = \frac{(1.6x10^{-19})(5500)(10^{17})(0.25x10^{-4})^2}{2\pi(13.1)(8.85x10^{-14})(10^{-4})^2}$$

or

$$\underline{f_T = 755\ GHz}$$

(b)

Saturation velocity model:

$$f_T = \frac{v_{sat}}{2\pi L}$$

Assuming $v_{sat} = 10^7\ cm/s$, we find

$$f_T = \frac{10^7}{2\pi(10^{-4})}$$

or

$$\underline{f_T = 15.9\ GHz}$$

13.36

(a) $V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$

where

$$V_{P2} = \frac{eN_d d_d^2}{2 \in_N} = \frac{(1.6x10^{-19})(3x10^{18})(350x10^{-8})^2}{2(12.2)(8.85x10^{-14})}$$

or

$$V_{P2} = 2.72\ V$$

Then

$$V_{off} = 0.89 - 0.24 - 2.72$$

or

$$\underline{V_{off} = -2.07\ V}$$

(b)

$$n_S = \frac{\in_N}{e(d + \Delta d)}(V_g - V_{off})$$

For $V_g = 0$, we have

$$n_S = \frac{(12.2)(8.85x10^{-14})}{(1.6x10^{-19})(350+80)\cdot 10^{-8}}(2.07)$$

or

$$\underline{n_S = 3.25x10^{12}\ cm^{-2}}$$

13.37

(a) We have

$$I_D(sat) = \frac{\in_N W}{(d+\Delta d)}\left(V_g - V_{off} - V_O\right)v_s$$

We find

$$\left(\frac{g_{mS}}{W}\right) = \frac{\partial}{\partial V_g}\left[\frac{I_D(sat)}{W}\right] = \frac{\in_N v_s}{(d+\Delta d)}$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)\left(2x10^7\right)}{(350+80)\cdot 10^{-8}} = 5.02\ \frac{S}{cm}$$

or

$$\frac{g_{mS}}{W} = 502\ \frac{mS}{mm}$$

(b)

At $V_g = 0$, we obtain

$$\frac{I_D(sat)}{W} = \frac{\in_N}{(d+\Delta d)}\left(-V_{off} - V_O\right)v_S$$

$$= \frac{(12.2)\left(8.85x10^{-14}\right)}{(350+80)\cdot 10^{-8}}(2.07-1)\left(2x10^7\right)$$

or

$$\frac{I_D(sat)}{W} = 5.37\ A/cm = 537\ mA/mm$$

13.38

$$V_{off} = \phi_B - \frac{\Delta E_C}{e} - V_{P2}$$

We want $V_{off} = -0.3\,V$, so

$$-0.30 = 0.85 - 0.22 - V_{P2}$$

or

$$V_{P2} = 0.93\,V = \frac{eN_d d_d^2}{2\in_N}$$

We can then write

$$d_d^2 = \frac{2\in_N V_{P2}}{eN_d}$$

$$= \frac{2(12.2)\left(8.85x10^{-14}\right)(0.93)}{\left(1.6x10^{-19}\right)\left(2x10^{18}\right)}$$

We then obtain

$$d_d = 2.51x10^{-6}\ cm = 251\ \mathring{A}$$