Chapter 14

Problem Solutions

14.1

(a) $\lambda = \dfrac{1.24}{E}\ \mu m$

Then

Ge: $E_g = 0.66\ eV \Rightarrow \underline{\lambda = 1.88\ \mu m}$

Si: $E_g = 1.12\ eV \Rightarrow \underline{\lambda = 1.11\ \mu m}$

GaAs: $E_g = 1.42\ eV \Rightarrow \underline{\lambda = 0.873\ \mu m}$

(b)

$$E = \frac{1.24}{\lambda}$$

For $\lambda = 570\ nm \Rightarrow \underline{E = 2.18\ eV}$

For $\lambda = 700\ nm \Rightarrow \underline{E = 1.77\ eV}$

14.2

(a) GaAs

$h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

so

$$\underline{\alpha \approx 1.5x10^4\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(1.5x10^4\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.59}$$

so the percent absorbed is (1-0.59), or
<u>41%</u>

(b) Silicon

Again $h\nu = 2\ eV \Rightarrow \lambda = 0.62\ \mu m$

So

$$\underline{\alpha \approx 4x10^3\ cm^{-1}}$$

Then

$$\frac{I(x)}{I_o} = \exp(-\alpha x) = \exp\left[-\left(4x10^3\right)\left(0.35x10^{-4}\right)\right]$$

or

$$\underline{\frac{I(x)}{I_o} = 0.87}$$

so the percent absorbed is (1-0.87), or
<u>13%</u>

14.3

$$g' = \frac{\alpha I(x)}{h\nu}$$

For $h\nu = 1.3\ eV \Rightarrow \lambda = \dfrac{1.24}{1.3} = 0.95\ \mu m$

For silicon, $\alpha \approx 3x10^2\ cm^{-1}$,

Then for

$I(x) = 10^{-2}\ W/cm^2$

we obtain

$$g' = \frac{\left(3x10^2\right)\left(10^{-2}\right)}{\left(1.6x10^{-19}\right)(1.3)} \Rightarrow$$

$$\underline{g' = 1.44x10^{19}\ cm^{-3}s^{-1}}$$

The excess concentration is

$$\delta n = g'\tau = \left(1.44x10^{19}\right)\left(10^{-6}\right) \Rightarrow$$

$$\underline{\delta n = 1.44x10^{13}\ cm^{-3}}$$

14.4

n-type GaAs, $\tau = 10^{-7}\ s$

(a)

We want

$$\delta n = \delta p = 10^{15}\ cm^{-3} = g'\tau = g'\left(10^{-7}\right)$$

or

$$g' = \frac{10^{15}}{10^{-7}} = 10^{22}\ cm^{-3}s^{-1}$$

We have

$$h\nu = 1.9\ eV \Rightarrow \lambda = \frac{1.24}{1.9} = 0.65\ \mu m$$

so that

$\alpha \approx 1.3x10^4\ cm^{-1}$

Then

$$g' = \frac{\alpha I(x)}{h\nu} \Rightarrow I(x) = \frac{(g')(h\nu)}{\alpha}$$

$$= \frac{\left(10^{22}\right)\left(1.6x10^{-19}\right)(1.9)}{1.3x10^4}$$

or

$$\underline{I(0) = 0.234\ W/cm^2 = I_o}$$

(b)

$$\frac{I(x)}{I_o} = 0.20 = \exp\left[-\left(1.3x10^4\right)x\right]$$

We obtain $\underline{x = 1.24\ \mu m}$

14.5
GaAs
(a)
For $h\nu = 1.65\ eV \Rightarrow \lambda = 0.75\ \mu m$
So
$$\alpha \approx 0.7x10^4\ cm^{-1}$$
For 75% obsorbed,
$$\frac{I(x)}{I_O} = 0.25 = \exp(-\alpha x)$$
Then
$$\alpha x = \ln\left(\frac{1}{0.25}\right) \Rightarrow x = \frac{1}{0.7x10^4}\ln\left(\frac{1}{0.25}\right)$$
or
$$\underline{x = 1.98\ \mu m}$$
(b)
For 75% transmitted,
$$\frac{I(x)}{I_O} = 0.75 = \exp\left[-\left(0.7x10^4\right)x\right]$$
we obtain
$$\underline{x = 0.41\ \mu m}$$

14.6
GaAs
For $x = 1\ \mu m = 10^{-4}\ cm$, we have 50% absorbed or 50% transmitted, then
$$\frac{I(x)}{I_O} = 0.50 = \exp(-\alpha x)$$
We can write
$$\alpha = \left(\frac{1}{x}\right)\cdot\ln\left(\frac{1}{0.5}\right) = \left(\frac{1}{10^{-4}}\right)\cdot\ln(2)$$
or
$$\alpha = 0.69x10^4\ cm^{-1}$$
This value corresponds to
$$\underline{\lambda = 0.75\ \mu m\ ,\ E = 1.65\ eV}$$

14.7
The ambipolar transport equation for minority carrier holes in steady state is
$$D_p\frac{d^2(\delta p_n)}{dx^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$
or
$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$
where $L_p^2 = D_p\tau_p$
The photon flux in the semiconductor is
$$\Phi(x) = \Phi_O\exp(-\alpha x)$$
and the generation rate is
$$G_L = \alpha\Phi(x) = \alpha\Phi_O\exp(-\alpha x)$$
so we have
$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = -\frac{\alpha\Phi_O}{D_p}\exp(-\alpha x)$$
The general solution is of the form
$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$
At $x \to \infty$, $\delta p_n = 0$
So that $B = 0$, then
$$\delta p_n = A\exp\left(\frac{-x}{L_p}\right) - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\exp(-\alpha x)$$
At $x = 0$, we have
$$D_p\frac{d(\delta p_n)}{dx}\bigg|_{x=0} = s\delta p_n\big|_{x=0}$$
so we can write
$$\delta p_n\big|_{x=0} = A - \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
and
$$\frac{d(\delta p_n)}{dx}\bigg|_{x=0} = -\frac{A}{L_p} + \frac{\alpha^2\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
Then we have
$$-\frac{AD_p}{L_p} + \frac{\alpha^2\Phi_O\tau_pD_p}{\alpha^2L_p^2 - 1} = sA - \frac{s\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}$$
Solving for A, we find
$$A = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left[\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\right]$$
The solution can now be written as
$$\delta p_n = \frac{\alpha\Phi_O\tau_p}{\alpha^2L_p^2 - 1}\cdot\left\{\frac{s + \alpha D_p}{s + \left(D_p/L_p\right)}\cdot\exp\left(\frac{-x}{L_p}\right) - \exp(-\alpha x)\right\}$$

14.8
We have

$$D_n \frac{d^2(\delta n_p)}{dx^2} + G_L - \frac{\delta n_p}{\tau_n} = 0$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where $L_n^2 = D_n \tau_n$

The general solution can be written in the form

$$\delta n_p = A\cosh\left(\frac{x}{L_n}\right) + B\sinh\left(\frac{x}{L_n}\right) + G_L \tau_n$$

For $s = \infty$ at $x = 0$ means that $\delta n_p(0) = 0$,
Then

$$0 = A + G_L \tau_n \Rightarrow A = -G_L \tau_n$$

At $x = W$,

$$-D_n \frac{d(\delta n_p)}{dx}\Big|_{x=W} = s_o \delta n_p \Big|_{x=W}$$

Now

$$\delta n_p(W) = -G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n$$

and

$$\frac{d(\delta n_p)}{dx}\Big|_{x=W} = -\frac{G_L \tau_n}{L_n}\sinh\left(\frac{W}{L_n}\right) + \frac{B}{L_n}\cosh\left(\frac{W}{L_n}\right)$$

so we can write

$$\frac{G_L \tau_n D_n}{L_n}\sinh\left(\frac{W}{L_n}\right) - \frac{BD_n}{L_n}\cosh\left(\frac{W}{L_n}\right)$$
$$= s_o\left[-G_L \tau_n \cosh\left(\frac{W}{L_n}\right) + B\sinh\left(\frac{W}{L_n}\right) + G_L \tau_n\right]$$

Solving for *B*, we find

$$B = \frac{G_L\left[L_n \sinh\left(\frac{W}{L_n}\right) + s_o \tau_n \cosh\left(\frac{W}{L_n}\right) - s_o \tau_n\right]}{\frac{D_n}{L_n}\cosh\left(\frac{W}{L_n}\right) + s_o \sinh\left(\frac{W}{L_n}\right)}$$

The solution is then

$$\delta n_p = G_L \tau_n\left[1 - \cosh\left(\frac{x}{L_n}\right)\right] + B\sinh\left(\frac{x}{L_n}\right)$$

where *B* was just given.

14.9

$$V_{OC} = V_t \ln\left(1 + \frac{J_L}{J_S}\right)$$
$$= (0.0259)\ln\left(1 + \frac{30x10^{-3}}{J_S}\right)$$

where

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

which becomes

$$J_S = (1.6x10^{-19})(1.8x10^6)^2 \times\left[\frac{1}{N_a}\cdot\sqrt{\frac{225}{5x10^{-8}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{7}{5x10^{-8}}}\right]$$

or

$$J_S = (5.18x10^{-7})\left[\frac{6.7x10^4}{N_a} + 1.18x10^{-15}\right]$$

Then

N_a	$J_S(A/cm^2)$	$V_{OC}(V)$
1E15	3.47E-17	0.891
1E16	3.47E-18	0.950
1E17	3.48E-19	1.01
1E18	3.53E-20	1.07

14.10
(a)

$$I_L = J_L \cdot A = (25x10^{-3})(2) = 50\ mA$$

We have

$$J_S = en_i^2\left[\frac{1}{N_a}\cdot\sqrt{\frac{D_n}{\tau_n}} + \frac{1}{N_d}\cdot\sqrt{\frac{D_p}{\tau_p}}\right]$$

or

$$J_S = (1.6x10^{-19})(1.5x10^{10})^2 \times\left[\frac{1}{3x10^{16}}\cdot\sqrt{\frac{18}{5x10^{-6}}} + \frac{1}{10^{19}}\cdot\sqrt{\frac{6}{5x10^{-7}}}\right]$$

which becomes

$$J_S = 2.29x10^{-12}\ A/cm^2$$

or

$$I_S = 4.58x10^{-12}\ A$$

We have

$$I = I_L - I_S\left[\exp\left(\frac{V}{V}\right) - 1\right]$$

or

$$I = 50x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

We see that when $I = 0$, $\underline{V = V_{OC} = 0.599\ V}$.

We find

$V(V)$	$I(mA)$
0	50
0.1	50
0.2	50
0.3	50
0.4	49.9
0.45	49.8
0.50	48.9
0.55	42.4
0.57	33.5
0.59	14.2

(b)
The voltage at the maximum power point is found from

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{50x10^{-3}}{4.58x10^{-12}} = 1.092x10^{10}$$

By trial and error,

$$\underline{V_m = 0.520\ V}$$

At this point, we find

$$\underline{I_m = 47.6\ mA}$$

so the maximum power is

$$P_m = I_m V_m = (47.6)(0.520)$$

or

$$\underline{P_m = 24.8\ mW}$$

(c)
We have

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{V_m}{I_m} = \frac{0.520}{47.6x10^{-3}}$$

or

$$\underline{R = 10.9\ \Omega}$$

14.11
If the solar intensity increases by a factor of 10, then I_L increases by a factor of 10 so that $I_L = 500\ mA$. Then

$$I = 500x10^{-3} - 4.58x10^{-12}\left[\exp\left(\frac{V}{V_t}\right) - 1\right]$$

At the maximum power point

$$\left[1 + \frac{V_m}{V_t}\right] \cdot \exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$$

$$= 1 + \frac{500x10^{-3}}{4.58x10^{-12}} = 1.092x10^{11}$$

By trial and error, we find

$$\underline{V_m = 0.577\ V}$$

and the current at the maximum power point is

$$\underline{I_m = 478.3\ mA}$$

The maximum power is then

$$\underline{P_m = I_m V_m = 276\ mW}$$

The maximum power has increased by a factor of 11.1 compared to the previous problem, which means that the efficiency has increased slightly.

14.12
Let $x = 0$ correspond to the edge of the space charge region in the p-type material. Then

$$D_n \frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{\tau_n} = -G_L$$

or

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{G_L}{D_n}$$

where

$$G_L = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$$

Then we have

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = -\frac{\alpha\Phi_O}{D_n}\exp(-\alpha x)$$

The general solution is of the form

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) + B\exp\left(\frac{+x}{L_p}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}\exp(-\alpha x)$$

At $x \to \infty$, $\delta n_p = 0$ so that $B = 0$, then

$$\delta n_p = A\exp\left(\frac{-x}{L_n}\right) - \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1} \cdot \exp(-\alpha x)$$

We also have $\delta n_p(0) = 0 = A - \dfrac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$,

which yields

$$A = \frac{\alpha\Phi_O\tau_n}{\alpha^2 L_n^2 - 1}$$

We then obtain

$$\delta n_p = \frac{\alpha\Phi_O \tau_n}{\alpha^2 L_n^2 - 1}\left[\exp\left(\frac{-x}{L_n}\right) - \exp(-\alpha x)\right]$$

where Φ_O is the incident flux at $x = 0$.

14.13

For 90% absorption, we have

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

Then

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

or

$$x = \left(\frac{1}{\alpha}\right)\cdot \ln(10)$$

For $h\nu = 1.7\ eV$, $\alpha \approx 10^4\ cm^{-1}$

Then

$$x = \left(\frac{1}{10^4}\right)\cdot \ln(10) \Rightarrow \underline{x = 2.3\ \mu m}$$

and for $h\nu = 2.0\ eV$, $\alpha \approx 10^5\ cm^{-1}$, so that

$\underline{x = 0.23\ \mu m}$

14.14

$G_L = 10^{20}\ cm^{-3}s^{-1}$ and $N_d > N_a$ so holes are the minority carrier.

(a)

$$\delta p = g'\tau = G_L \tau_p$$

so that

$$\delta p = \delta n = (10^{20})(10^{-7})$$

or

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$
$$= (1.6x10^{-19})(10^{13})(1000 + 430)$$

or

$$\underline{\Delta\sigma = 2.29x10^{-3}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = \frac{(\Delta\sigma)AV}{L}$$
$$= \frac{(2.29x10^{-3})(10^{-3})(5)}{100x10^{-4}}$$

or

$$\underline{I_L = 1.15\ mA}$$

(d)

The photoconductor gain is

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right)$$

where

$$t_n = \frac{L}{\mu_n \mathrm{E}} = \frac{L^2}{\mu_n V}$$

Then

$$\Gamma_{ph} = \frac{\tau_p \mu_n V}{L^2}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}(\mu_n + \mu_p)$$

or

$$\Gamma_{ph} = \frac{(10^{-7})(5)}{(100x10^{-4})^2}(1000 + 430)$$

or

$$\underline{\Gamma_{ph} = 7.15}$$

14.15

n-type, so holes are the minority carrier

(a)

$$\delta p = G_L \tau_p = (10^{21})(10^{-8})$$

so that

$$\underline{\delta p = \delta n = 10^{13}\ cm^{-3}}$$

(b)

$$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$$
$$= (1.6x10^{-19})(10^{13})(8000 + 250)$$

or

$$\underline{\Delta\sigma = 1.32x10^{-2}\ (\Omega - cm)^{-1}}$$

(c)

$$I_L = J_L \cdot A = (\Delta\sigma)A\mathrm{E} = \frac{(\Delta\sigma)AV}{L}$$
$$= \frac{(1.32x10^{-2})(10^{-4})(5)}{100x10^{-4}}$$

or $\underline{I_L = 0.66\ mA}$

(d)

$$\Gamma_{ph} = \frac{\tau_p}{t_n}\left(1 + \frac{\mu_p}{\mu_n}\right) = \frac{\tau_p V}{L^2}(\mu_n + \mu_p)$$
$$= \frac{(10^{-8})(5)}{(100x10^{-4})^2}(8000 + 250)$$

or $\underline{\Gamma_{ph} = 4.13}$

14.16

$\Phi(x) = \Phi_O \exp(-\alpha x)$

The electron-hole generation rate is

$g' = \alpha\Phi(x) = \alpha\Phi_O \exp(-\alpha x)$

and the excess carrier concentration is

$\delta p = \tau_p \alpha\Phi(x)$

Now

$\Delta\sigma = e(\delta p)(\mu_n + \mu_p)$

and

$J_L = \Delta\sigma \mathrm{E}$

The photocurrent is now found from

$$I_L = \iint \Delta\sigma \mathrm{E} \cdot dA = \int_0^W dy \int_0^{x_O} \Delta\sigma \mathrm{E} \cdot dx$$

$$= We(\mu_n + \mu_n)\mathrm{E}\int_0^{x_O} \delta p \cdot dx$$

Then

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O \tau_p \int_0^{x_O} \exp(-\alpha x)dx$$

$$= We(\mu_n + \mu_p)\mathrm{E}\alpha\Phi_O \tau_p \left[-\frac{1}{\alpha}\exp(-\alpha x)\right]_0^{x_O}$$

which becomes

$$I_L = We(\mu_n + \mu_p)\mathrm{E}\Phi_O \tau_p [1 - \exp(-\alpha x_O)]$$

Now

$$I_L = (50x10^{-4})(1.6x10^{-19})(1200 + 450)(50)$$
$$\times (10^{16})(2x10^{-7})[1 - \exp(-(5x10^4)(10^{-4}))]$$

or

$$\underline{I_L = 0.131\ \mu A}$$

14.17

(a)

$$V_{bi} = (0.0259)\ln\left[\frac{(2x10^{16})(10^{18})}{(1.5x10^{10})^2}\right] = 0.832\ V$$

The space charge width is

$$W = \left[\frac{2\in(V_{bi} + V_R)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85x10^{-14})(0.832 + 5)}{1.6x10^{-19}} \times \left(\frac{2x10^{16} + 10^{18}}{(2x10^{16})(10^{18})}\right)\right]^{1/2}$$

or

$W = 0.620\ \mu m$

The prompt photocurrent density is

$$J_{L1} = eG_L W = (1.6x10^{-19})(10^{21})(0.620x10^{-4})$$

or

$$\underline{J_{L1} = 9.92\ mA/cm^2}$$

(b)

The total steady-state photocurrent density is

$$J_L = e(W + L_n + L_p)G_L$$

We find

$$L_n = \sqrt{D_n \tau_n} = \sqrt{(25)(2x10^{-7})} = 22.4\ \mu m$$

and

$$L_p = \sqrt{D_p \tau_p} = \sqrt{(10)(10^{-7})} = 10.0\ \mu m$$

Then

$$J_L = (1.6x10^{-19})(0.62 + 22.4 + 10.0)(10^{-4})(10^{21})$$

or

$$\underline{J_L = 0.528\ A/cm^2}$$

14.18

In the n-region under steady state and for $\mathrm{E} = 0$, we have

$$D_p \frac{d^2(\delta p_n)}{dx'^2} + G_L - \frac{\delta p_n}{\tau_p} = 0$$

or

$$\frac{d^2(\delta p_n)}{dx'^2} - \frac{\delta p_n}{L_p^2} = -\frac{G_L}{D_p}$$

where $L_p^2 = D_p \tau_p$ and where x' is positive in the negative x direction. The homogeneous solution is found from

$$\frac{d^2(\delta p_{nh})}{dx'^2} - \frac{\delta p_{nh}}{L_p^2} = 0$$

The general solution is found to be

$$\delta p_{nh} = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right)$$

The particular solution is found from

$$\frac{-\delta p_{np}}{L_p^2} = \frac{-G_L}{D_p}$$

which yields

$$\delta p_{np} = \frac{G_L L_p^2}{D_p} = G_L \tau_p$$

The total solution is the sum of the homogeneous and particular solutions, so we have

$$\delta p_n = A\exp\left(\frac{-x'}{L_p}\right) + B\exp\left(\frac{+x'}{L_p}\right) + G_L\tau_p$$

One boundary condition is that δp_n remains finite as $x' \to \infty$ which means that $B = 0$. Then At $x' = 0$, $p_n(0) = 0 = \delta p_n(0) + p_{nO}$, so that

$$\delta p_n(0) = -p_{nO}$$

We find that

$$A = -(p_{nO} + G_L\tau_p)$$

The solution is then written as

$$\delta p_n = G_L\tau_p - (G_L\tau_p + p_{nO})\exp\left(\frac{-x'}{L_p}\right)$$

The diffusion current density is found as

$$J_p = -eD_p \frac{d(\delta p_n)}{dx}\Big|_{x'=0}$$

But

$$\frac{d(\delta p_n)}{dx} = -\frac{d(\delta p_n)}{dx'}$$

since x and x' are in opposite directions. So

$$J_p = +eD_p \frac{d(\delta p_n)}{dx'}\Big|_{x'=0}$$

$$= eD_p\left[-(G_L\tau_p + p_{nO})\right]\left(\frac{-1}{L_p}\right)\exp\left(\frac{-x'}{L_p}\right)\Big|_{x'=0}$$

Then

$$\underline{J_p = eG_L L_p + \frac{eD_p p_{nO}}{L_p}}$$

14.19

We have

$$J_L = e\Phi_O[1 - \exp(-\alpha W)]$$

$$= (1.6x10^{-19})(10^{17})[1 - \exp(-(3x10^3)W)]$$

or

$$J_L = 16[1 - \exp(-(3x10^3)W)]\ (mA)$$

Then for $W = 1\ \mu m = 10^{-4}\ cm$, we find

$$\underline{J_L = 4.15\ mA}$$

For $W = 10\ \mu m \Rightarrow \underline{J_L = 15.2\ mA}$

For $W = 100\ \mu m \Rightarrow \underline{J_L = 16\ mA}$

14.20

The minimum α occurs when $\lambda = 1\ \mu m$ which gives $\alpha = 10^2\ cm^{-1}$. We want

$$\frac{\Phi(x)}{\Phi_O} = \exp(-\alpha x) = 0.10$$

which can be written as

$$\exp(+\alpha x) = \frac{1}{0.1} = 10$$

Then

$$x = \frac{1}{\alpha}\ln(10) = \frac{1}{10^2}\ln(10)$$

or

$$\underline{x = 230\ \mu m}$$

14.21

For the $Al_xGa_{1-x}As$ system, a direct bandgap for $0 \le x \le 0.45$, we have

$$E_g = 1.424 + 1.247x$$

At $x = 0.45$, $E_g = 1.985\ eV$, so for the direct bandgap

$$\underline{1.424 \le E_g \le 1.985\ eV}$$

which yields

$$\underline{0.625 \le \lambda \le 0.871\ \mu m}$$

14.22

For $x = 0.35$ in $GaAs_{1-x}P_x$, we find

(a) $\underline{E_g = 1.85\ eV}$ and (b) $\underline{\lambda = 0.670\ \mu m}$

14.23

(a)

For GaAs, $\bar{n}_2 = 3.66$ and for air, $\bar{n}_1 = 1.0$.

The critical angle is

$$\theta_C = \sin^{-1}\left(\frac{\bar{n}_1}{\bar{n}_2}\right) = \sin^{-1}\left(\frac{1}{3.66}\right) = 15.9°$$

The fraction of photons that will not experience total internal reflection is

$$\frac{2\theta_C}{360} = \frac{2(15.9)}{360} \Rightarrow \underline{8.83\%}$$

(b)

Fresnel loss:

$$R = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2 = \left(\frac{3.66 - 1}{3.66 + 1}\right)^2 = 0.326$$

The fraction of photons emitted is then

$$(0.0883)(1 - 0.326) = 0.0595 \Rightarrow \underline{5.95\%}$$

14.24
We can write the external quantum efficiency as

$\eta_{ext} = T_1 \cdot T_2$

where $T_1 = 1 - R_1$ with R_1 is the reflection coefficient (Fresnel loss), and the factor T_2 is the fraction of photons that do not experience total internal reflection. We have

$$R_1 = \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

so that

$$T_1 = 1 - R_1 = 1 - \left(\frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1}\right)^2$$

which reduces to

$$T_1 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}$$

Now consider a solid angle from the source point. The surface area described by the solid angle is πp^2. The factor T_1 is given by

$$T_1 = \frac{\pi p^2}{4\pi R^2}$$

From the geometry, we have

$$\sin\left(\frac{\theta_C}{2}\right) = \frac{p/2}{R} \Rightarrow p = 2R\sin\left(\frac{\theta_C}{2}\right)$$

Then the area is

$$A = \pi p^2 = 4R^2\pi\sin^2\left(\frac{\theta_C}{2}\right)$$

Now

$$T_1 = \frac{\pi p^2}{4\pi R^2} = \sin^2\left(\frac{\theta_C}{2}\right)$$

From a trig identity, we have

$$\sin^2\left(\frac{\theta_C}{2}\right) = \frac{1}{2}(1 - \cos\theta_C)$$

Then

$$T_1 = \frac{1}{2}(1 - \cos\theta_C)$$

The external quantum efficiency is now

$$\eta_{ext} = T_1 \cdot T_2 = \frac{4\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2} \cdot \frac{1}{2}(1 - \cos\theta_C)$$

or

$$\underline{\eta_{ext} = \frac{2\bar{n}_1\bar{n}_2}{(\bar{n}_1 + \bar{n}_2)^2}(1 - \cos\theta_C)}$$

14.25
For an optical cavity, we have

$$N\left(\frac{\lambda}{2}\right) = L$$

If λ changes slightly, then N changes slightly also. We can write

$$\frac{N_1\lambda_1}{2} = \frac{(N_1 + 1)\lambda_2}{2}$$

Rearranging terms, we find

$$\frac{N_1\lambda_1}{2} - \frac{(N_1 + 1)\lambda_2}{2} = \frac{N_1\lambda_1}{2} - \frac{N_1\lambda_2}{2} - \frac{\lambda_2}{2} = 0$$

If we define $\Delta\lambda = \lambda_1 - \lambda_2$, then we have

$$\frac{N_1}{2}\Delta\lambda = \frac{\lambda_2}{2}$$

We can approximate $\lambda_2 = \lambda$, then

$$\frac{N_1\lambda}{2} = L \Rightarrow N_1 = \frac{2L}{\lambda}$$

Then

$$\frac{1}{2} \cdot \frac{2L}{\lambda}\Delta\lambda = \frac{\lambda}{2}$$

which yields

$$\underline{\Delta\lambda = \frac{\lambda^2}{2L}}$$

14.26
For GaAs,

$$h\nu = 1.42\ eV \Rightarrow \lambda = \frac{1.24}{E} = \frac{1.24}{1.42}$$

or

$$\lambda = 0.873\ \mu m$$

Then

$$\Delta\lambda = \frac{\lambda^2}{2L} = \frac{(0.873x10^{-4})^2}{2(0.75x10^{-4})} = 5.08x10^{-7}\ cm$$

or

$$\underline{\Delta\lambda = 5.08x10^{-3}\ \mu m}$$
